



Page NO.

1. 1 to 73 – Mid Term
2. 74 to 174 – Final Term

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25<sup>th</sup> BATCH

COMPUTER AND COMMUNICATION ENGINEERING

International Islamic University Chittagong

**COURSE CODE: PHY-1201**

**COURSE TITLE: Physics II**

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Lecturer in Physics

Computer & Communication  
Engineering

Physics - Note - 01

1st semester:-

Mid term + Final

 Tuesday  
16.07.2020



Date: 18.07.20

Saturday

Lecture - 01 Lesson - 1

Gravity and Gravitation

⊗ Aristotle believed & said that, "a heavier body to reach the ground is less as compared to a lighter body dropped from the same height."

In 1590, Galileo disapproved the idea. He did a public experiment in "Pisa" tower with a stone and a paper. He said that due to a resistance offered by air on the paper, the paper reached after the stone. If that experiment was done in vacuum then both stone and paper would reach on the ground floor at the same time.

## Kepler's law of planetary motion:-

1] Every planet moves in an elliptical orbit with the sun being one of its foci (Focus is plural)

2] The radius vector drawn from the sun to the planet sweeps out equal areas in equal interval of time.

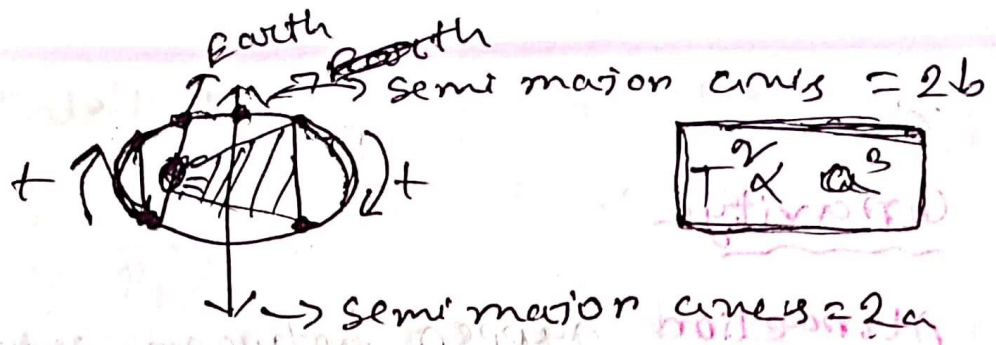
[ সমান সময়ে সমান দূরত্ব বা ক্ষেত্রফল অতিক্রম করে ] => [ Laws of areas ]

3] [The periods of planets]:-

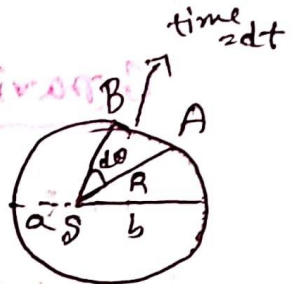
The square of the periods of revolution of a planet around the sun is proportional to the cube of the semi major axis of the orbit.

[ সময় কোণের বর্গ তার অক্ষের ঘনত্ব দ্বারা  
সমান তখন দূরত্ব সমান, আর সময় গুরু  
দূরত্ব দ্বারা সমান অক্ষের ঘনত্ব ]





Formula of Kepler's law no. 2 :-



Area swept in time  $\Delta t$

$$\therefore \text{Area ABS} = \frac{1}{2} R \times R d\theta = \frac{1}{2} R^2 d\theta$$

$\propto a \cdot b$

Area velocity,

$$\frac{\frac{1}{2} R^2 d\theta}{dt} = \frac{1}{2} R^2 \left( \frac{d\theta}{dt} \right) = \frac{1}{2} R^2 \omega = \text{Constant}$$

If we so,  $\frac{1}{2} R^2 \omega$  is constant

as a result,  $\frac{1}{2} M R^2 \omega$  is also a constant

Total area of ellipse

$$= \pi a b \quad \left[ \begin{array}{l} a \text{ is semi minor axis} \\ b \text{ is semi major axis} \end{array} \right]$$

एक वर्ष का काल कोला द्वारा घूर्णित समय,

$$T = \frac{\text{Area}}{\text{Area velocity}} = \frac{\pi a b}{\frac{1}{2} R^2 \omega} = \frac{2 \pi a b}{R^2 \omega}$$

3]

Gravity:-

Attraction happen between earth & many other objects.

Gravitation:-

The attraction force between any two bodies in the universe is called gravitation.

Derivation of law of gravitation:-

(माध्यमकर्मन आर्शन आविष्कार)

$$F_1 = M_1 R_1 \omega_1^2 = M_1 R_1 \left[ \frac{2\pi}{T_1} \right]^2 \rightarrow \text{Force of Planet A} \quad \text{--- (i)}$$

$$F_2 = M_2 R_2 \omega_2^2 = M_2 R_2 \left[ \frac{2\pi}{T_2} \right]^2 \rightarrow \text{Force of Planet B} \quad \text{--- (ii)}$$

(i) : (ii)

$$\frac{F_1}{F_2} = \frac{M_1 R_1 \left[ \frac{2\pi}{T_1} \right]^2}{M_2 R_2 \left[ \frac{2\pi}{T_2} \right]^2} = \left( \frac{M_1}{M_2} \right) \left( \frac{R_1}{R_2} \right) \left( \frac{T_2}{T_1} \right)^2 \quad \text{--- (iii)}$$



Kepler's third law

$$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{R_2}{R_1}\right)^3$$

Substituting this (i) in equation (ii)

$$\frac{F_1}{F_2} = \left(\frac{M_1}{M_2}\right) \left(\frac{R_2}{R_1}\right)^2$$

$$\frac{F_1 R_1^2}{M_1} = \frac{F_2 R_2^2}{M_2} = \text{Constant}$$

$$\therefore F \propto \frac{M}{R^2} \text{ or, (i) } F \propto M \text{ \& (ii) } F \propto \frac{1}{R^2}$$

এখানে দূরত্ব বৃদ্ধিতে

(-) কমে

According to the Newton:-

The force of attraction must be mutual, a force exerted by <sup>a</sup> earth on the body. body on the earth must be equal and opposite to the force exerted by the earth on the body. Newton reduced the

Kepler's laws into a single law of gravitation.

পৃথিবী নিউটনের মতে, বলের আকর্ষণ দূর দিক শূন্যে সমান হতে হবে। পৃথিবীর উন্নত কোনো কিছু আকর্ষণ বলের মান, পৃথিবী হতে ওই বস্তু উন্নত দূরত্বের বিপরীত বলও সমান হতে হবে।

☐ Newton's Universal law of gravitation  
(নিউটনের সর্বজনীন সূত্র):-

This law is true not only for the heavenly bodies but also for any two bodies of the Universe.

Law:-

The attraction force between any two bodies in the universe is directly proportional to the ~~square~~ of the product of their masses & inversely proportional to the square of the distance between



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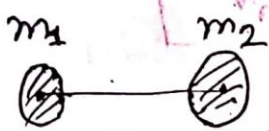
(নিউটনের মহাকর্ষ সূত্র):-

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Law:-

The attraction force between any two bodies in the universe is directly proportional to the square of the product of their masses & inversely proportional to the square of the distance between

them  $F = \frac{G M m}{R^2}$



$R \rightarrow$  मध्यवर्ती दूरी

all to be in same direction

Fd M.m

Or,  $F \propto \frac{1}{R^2}$  (अर्थात्  $F \propto \frac{Mm}{R^2}$ )

$F = G \frac{Mm}{R^2}$  [  $G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$  ]

# According to the Newton's second law

of motion  $F = ma$ .

Here  $a = g$ , where  $g$  is the acceleration due to gravity.

$\frac{g'}{g} = \frac{r^2}{R^2}$  [  $g' =$  ग्रहण अतिकर्षक त्वरण  
 $g =$  पृथिवी अतिकर्षक त्वरण ]  
 $r =$  पृथिवी व्यासार्ध  
 $R =$  ग्रहण व्यासार्ध ]

Or,  $g' = \frac{g r^2}{R^2}$

Or,  $g' = \frac{v^2}{R^2} = \frac{\left(\frac{2\pi R}{T}\right)^2}{R^2} = \frac{4\pi^2 R}{T^2}$

$\therefore \frac{g r^2}{R^2} = \frac{4\pi^2 R}{T^2}$



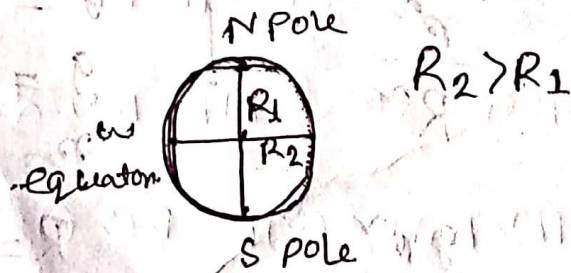
$$\text{or, } R^3 = \frac{g p^2 T^2}{4\pi^2} \quad \text{or, } R = \left[ \frac{g p^2 T^2}{4\pi^2} \right]^{1/3}$$



value of 'g' at the poles and at the equator :-

(মেরু ও অক্ষরেখায় 'g' এর মান)

⇒ The shape of the earth is slightly ellipsoidal (অঈকাকৃতি). It is bulging (গাঠিত) at the equator (অক্ষরেখায়) and flattened (চ্যুত) at the poles (মেরু). Its equatorial (নিরক্ষীয়) radius is more than the polar radius.

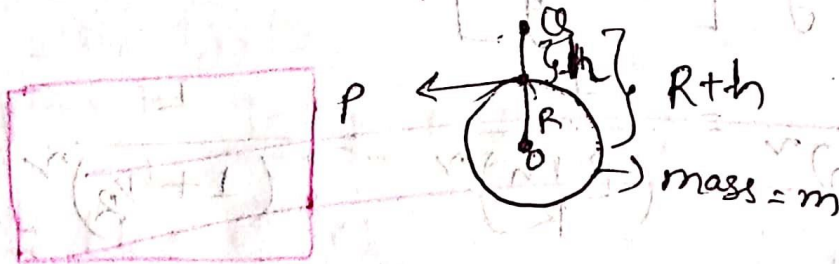


$$\therefore g = \frac{GM}{R^2}$$

$$\therefore g \propto \frac{1}{R^2}$$

Q variation of  $g$  with altitude :-

(পৃথিবী পৃষ্ঠের উচ্চতা দিয়ে কোথাও হোল  $g$  এর মান মেতাবে পরিবর্তিত হয়) :-



A body of mass ( $m$ ) when placed at the point  $P$  experiences a force ( $mg$ ) towards the centre of the earth.

$$mg = \frac{GMm}{R^2} \quad \text{--- (i)} \quad \left[ \because F = \frac{GMm}{R^2} \right]$$

$$F = mg$$

When the body is at  $Q$ , let the acceleration due to gravity be  $g'$ ,

$$\therefore mg' = \frac{GMm}{(R+h)^2} \quad \text{(ii)}$$

$$(ii) \div (i)$$

$$0 = \dots$$



$$\begin{aligned}
 \frac{g'}{g} &= \frac{GM}{(R+h)^2} \bigg/ \frac{GM}{R^2} \\
 &= \frac{R^2}{(R+h)^2} = \frac{1}{\left(\frac{R+h}{R}\right)^2} = \frac{1}{\left(1+\frac{h}{R}\right)^2}
 \end{aligned}$$

— শেষ

$\Rightarrow$  দৃষ্টিতে হতে কোনো বস্তুকে মত উদ্ভাও  
 নেওয়া যায়,  $h$  এর মান ততই কমতে থাকে।  
 কমতে কমতে 0 এর মধ্যে চলে আসে।

$$\therefore \left(\frac{h}{R}\right)^n = 0$$

$$\therefore \boxed{g' = g \left(1 - \frac{2h}{R}\right)} \quad \text{— শেষ}$$

Therefore the acceleration due to gravity decreases with altitude.

also Note:-

द्विपदी विस्तार :-

$$(1+x)^n = 1 - 2x + 3x^2 - 4x^3$$

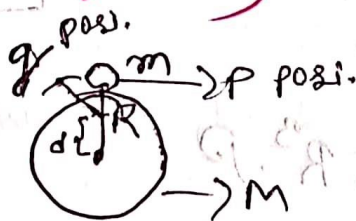
अतएव,

$$\left(1 + \frac{h}{R}\right)^2 = 1 - 2\frac{h}{R} + \frac{3h^2}{R^2} - \dots$$

$$\therefore \left(1 + \frac{h}{R}\right) = \left(1 - 2\frac{h}{R}\right) \text{ [क्योंकि अक्षरों के मान 0 हैं]}$$

variation of g with Depth:-

(गोलीयताय g का मान)



Let  $g$ , &  $g'$  be the accelerations due to gravity at P & Q respectively. At P the whole mass of the earth attracts the body & at Q it is attracted by the



Given mass of the earth of radius  $(R-h)$

$$\therefore mg = \frac{GMm}{R^2}$$

$$\text{or, } g = \frac{GM}{R^2}$$

Same,

$$mg' = \frac{GM'm}{(R-h)^2}$$

$$\text{or, } g' = \frac{GM'}{(R-h)^2}$$

[  $M'$  এর কারণ হলো  
 স্থানিক ভেতরে কোলে  
 একদিকে ব্যাসার্ধ কমলে  
 চতুর্দিকেই কম মাস।  
 আয়তন ছোট হলে  $g$   
 ও ছোট হয় ]

○

∴  $M' = \rho V$

$$\text{or, } M' = \rho V$$

at point P,

$$M' = \frac{4}{3} \pi R^3 \rho$$



at point Q,

$$M' = \frac{4}{3} \pi (R-h)^3 \rho$$

$$\therefore g = \frac{GM}{R^2} = \frac{G \cdot \frac{4}{3} \pi R^3 \cdot \rho}{R^2} = \frac{G \cdot 4 \cdot \pi R \cdot \rho}{3} \quad \text{--- (3)}$$

$\rho =$  ঘনত্ব  
(পৃথিবীর)

$$\therefore g' = \frac{GM'}{(R-d)^2} = \frac{\frac{4}{3} \pi (R-d)^3 \cdot \rho}{(R-d)^2} = \frac{4 \pi (R-d) \rho}{3} \quad \text{--- (4)}$$

#  $\rho$  is the Density of the earth

(4) ÷ (3)

$$\frac{g'}{g} = \frac{4 \pi G R \rho (R-d) \cdot \rho}{3} \times \frac{3}{4 \pi G R \rho}$$

$$= \frac{R-d}{R} = 1 - \frac{d}{R}$$

$$\therefore g' = g \left(1 - \frac{d}{R}\right) \quad \left[ \because g \ll R \right]$$

$$g' = g (0.9 \dots)$$

$g' < g$  (এর মানে হলো, পৃথিবীর যত হাড়ীতায় যাওয়া যায় ততই  $g'$  এর মান কমবে)

যদি  $d = R$  হলে  $g' = 0$



□ হাভীয়ে হোল্ডে চ ব্যাসার্ধ কমে, কিন্তু তাও

অধিকমত স্বরণের মান কমে কোলা?

$$\Rightarrow \frac{GM}{R^2} = \frac{G \cdot \frac{4}{3} \cdot \pi R^3 \cdot \rho}{R^2}$$

এখানে, তর এত মান কমে  $R^3$  আকায়ে আর  
 ব্যাসার্ধ কমে  $R^2$  আকায়ে। সেয়ে তর স্বরণ বেছি  
 কমে, তাই অধিকমত স্বরণের মান কমে।

□ variation of  $g$  with rotation of the earth :-

$$g' = g (1 - \omega^2 R \cos^2 \delta)$$

$\omega$  = কৌনিক বেগ

$R$  = পৃ. ব্যাসার্ধ

$\cos \delta$  = কেন্দ্রাভিমুখ  
 বলের কোণ

$g$  = অধিকমত স্বরণ

$g'$  = পরিবর্তিত

কিম্বদ ত্রুখা বরাবর,  $\delta = 90^\circ$ ;

অর্থাৎ,  $\cos \delta = \cos 90^\circ = 0$

$$\therefore g' = g - \omega^2 R \Rightarrow \text{কিম্বদ ত্রুখা}$$

মেঝে অথলে,

$$\lambda = 90^\circ; \text{ অর্থাৎ, } \cos \lambda = 0.$$

$$\therefore g_{90} = g.$$

আংশিক হাতিয় ফলে,

বিমুখ অথলে চ্যুত এর মান স্বাভাবিক কম  
মেঝে " (নিকট, দূরত্ব) " (বলি)

এর ফলে, আংশিক হাতিয় ফলে বম্বুর ওজন বিমুখ  
অথলে হতে মেঝে অথলের দিকে ক্রম  
বৃদ্ধি লায়।

Gravitational Field:-

(মহাকর্ষীয় ক্ষেত্র)

The gravitational force of attraction is perceived (অনুভূত) is called gravitational field.

If the gravitational field at a point is  $F$ , the force acting on a mass  $m$  is  $F$ .



$$\therefore F = ma = mE$$

$$\text{or, } E = \frac{F}{m} \quad \text{--- (i)}$$

$$= -\frac{dV}{dr} \quad \left[ \text{also defined as the negative gradient of gravitational potential} \right]$$

## Gravitational potential

(मशकरीय जम्हाना)

Consider a particle A of mass  $m$ ,  $P$  is a point at a distance  $r$  from A.

The gravitational intensity at  $P$ .

$$E = \frac{F}{m} = \frac{GM}{r^2} \quad \text{--- (i)}$$



$$E = -\frac{dV}{dr} \quad \text{--- (ii)}$$

$$\text{or, } dV = -E \times dr$$

$$= -\left(\frac{GM}{r^2}\right) dr \quad \text{--- [(ii) शुरुवात]}$$

~~A~~ Infinity to Integration between the limits of infinity and  $r$ ,

$$v = -\frac{GM}{r}$$

∴ The gravitational potential at a point due to a point mass,


$$= \left( -\frac{GM}{r} \right)$$

Q Gravitational potential energy of a body depends upon:-

(i) The mass of the body at A.

(ii) The mass of the body at B

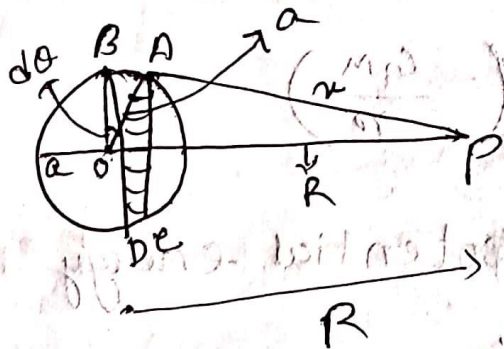
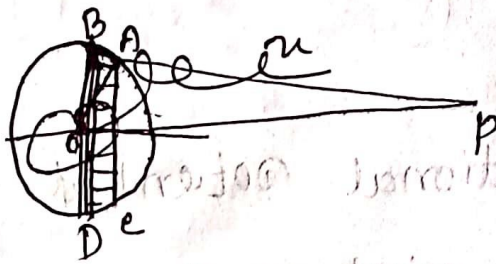
(iii) the distance between two masses.

\*\*\*  Gravitational potential & field at a point due to a spherical shell:-  
( $\vec{g}$ )

⇒

P.T.O





$2\pi a^2 \sin\theta$

$AB = a \, d\theta$  [  $\because s = r\theta$  ]

Surface area of element AC, BD

$= 2\pi (a \sin\theta) a \, d\theta$

$= 2\pi a^2 \sin\theta \, d\theta$  — (2)

Mass of the element,

$m = (2\pi a^2 \sin\theta \, d\theta) \rho$  — (3)

Now,

$AP^2 = r^2$

P, point of gravitational potential,

$$dv = - \frac{Edm}{r^2} \quad m, \quad E = \frac{dv}{dr}$$

$$\therefore E = \frac{GMm}{r^2 m} = \frac{GM}{r^2}$$

$$\therefore dv = - \frac{GM}{r^2}$$

$$= - G \frac{2\pi a^2 \sin\theta d\theta}{r} \quad \text{--- (4)}$$

$\Delta AOP$  এর জন্য,

$$r^2 = a^2 + R^2 - 2aR \cos\theta$$

এখন, Differentiation করে পাই,

$$2r dr = 2aR \sin\theta d\theta$$

$$\text{সুতরাং, } r = \frac{aR \sin\theta d\theta}{dr} \quad \text{--- (5)}$$

(4) নং equation - এ  $r$  এর মান বসাই,

$$dv = \frac{2\pi G a^2 \sin\theta d\theta}{aR \sin\theta d\theta} dr$$

$$\therefore dv = - \left( \frac{2\pi G a^2}{R} \right) \quad \text{--- (6)}$$

$$v = - \int_{R+a}^{R+a} \frac{2\pi G a^2}{R} dr = \frac{2\pi G a^2}{R} [R+a - R+a]$$



$$v = - \frac{4\pi a^3 \rho}{R} \quad \text{--- (7)}$$

আমরা জানি,

$$\text{তবে, } M = 4\pi a^3 \rho$$

$$\therefore v = - \frac{GM}{R} \quad \text{--- (8)}$$

$$F = - \frac{dv}{dR} = - \frac{d}{dR} \left[ - \frac{GM}{R} \right]$$

$$= - \frac{GM}{R^2}$$

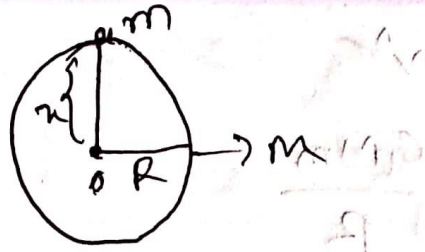
$$\therefore \vec{F} = - \frac{GM\vec{R}}{R^3}$$

মুক্তিবেগ

(Escape velocity)

Escape velocity is defined as the velocity with which a body has to be projected (vertically upwards from the earth's gravitational field altogether

(যদি কোনো বস্তুকে gravitational field এর বাইরে নিয়ে যাওয়া হয়)



$R$  = Radius of the earth

$M$  = mass of the earth

$m$  = mass of the object

$$F = \frac{GMm}{r^2} \quad (1)$$

$$dW = F \cdot dr$$

$$dW = \left( \frac{GMm}{r^2} \right) dr \quad (2)$$

To take the body from earth's surface to infinity we will get total work ( $W$ ).

$$W = \int_{\infty}^R F dr = \int_{\infty}^R \frac{GMm}{r^2} dr = \int_R^{\infty} \frac{GMm}{r^2} dr$$

$$= GMm \int_R^{\infty} \frac{1}{r^2} dr$$

$$= GMm \left[ -\frac{1}{r} \right]_R^{\infty}$$

$$= GMm \left[ -\frac{1}{\infty} + \frac{1}{R} \right]$$

$$= \left[ -\frac{GMm}{r} \right]_R^{\infty} = \left[ \frac{GMm}{R} \right]$$

$$\therefore \text{K.E} = \frac{1}{2}mv^2$$

$$\therefore \frac{1}{2}mv^2 = \frac{GMm}{R}$$

$$\therefore v = \sqrt{\frac{2GM}{R}}$$



Also at the surface of the earth,

$$mg = \frac{GMm}{R^2}$$

$$\therefore GM = gR^2$$

Substituting the value of  $GM$  in equation (iv)

$$v = \sqrt{2gR}$$

$\therefore$  The value of  $g$  at the earth's surface =  $9.8 \text{ m/s}^2$

$$\text{and } R = 6 \times 10^6 \text{ m}$$

$$v = \sqrt{2 \times 9.8 \times 64 \times 10^6}$$

$$\therefore v = 11.2 \times 10^6 \text{ m/s}^2$$

$$= 11.2 \text{ km/s}^2$$



Date:- 06.08.20  
Wednesday 7:55 PM

## Lecture - 02

### Dynamic of rigid bodies

Dynamic :- (शक्तिविद्युत)

Relating to forces producing motion

The part of mechanics in which motions to the forces associated with it & the proportion of moving object is related is known as dynamics.

Rigid Body (अनमनीय कणिका/वस्तु)

A body is said to be rigid when it is impossible to change its shape by the application of a force however large. For practical purpose all solid bodies are considered as rigid bodies.

Short Form :-

where two points of a body are fixed or unmovable but though the whole body is



movable:

Lecture - 05



Examples of rigid bodies

Examples: (continued)

The difference between two points of this body can't move though the body is movable.

Displacement (स्थापना) :-

Displacement is defined as the shortest distance between initial & final position of a body in a particular direction.



## velocity (वेग) [Velocity (वेग)]

Velocity is defined as the rate of change of displacement with respect to time. If  $\Delta s$  displacement changes with time  $\Delta t$ ,

∴ velocity,  $v = \frac{\Delta s}{\Delta t}$  [ $\text{ms}^{-1}$ ]

∴ acceleration,  $a = \frac{\Delta v}{\Delta t}$  [ $\text{ms}^{-2}$ ]

## Instantaneous velocity (तत्कालिक वेग) :-

It is the limit of the average velocity as the elapsed (उत्तिका) approaches zero, or the derivative of  $(s)$

with respect to  $t$ :

$$\text{inst } v = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

$$\therefore \text{Ins. acceleration, } a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

⇒ Like average velocity, instantaneous velocity is a vector with dimension of length per  $t$ .



□ ভরবেগ (mass velocity) :-

(Momentum)

ভর ও বেগের গুণফলকে ভরবেগ বলে,

$$p = mv$$

□ Angular Displacement :- ( $\theta$ )

The angle subtended (সম্মুখ) in a particular time interval by an object or particle at a center of circular path (বৃত্তাকার পথ) along which it is revolving (ঘুরতে থাকে) is called Angular Displacement.



$$\theta = \frac{s}{r} \text{ বা, } \boxed{s = r\theta}$$

এখানে  $\theta$  এর মান সমস্তই রেডিয়ানে

নেওয়া হয়।

রেডিয়ান:-

কোনো বৃত্তের ব্যাসার্ধের সমান চাপ বৃত্তের কেন্দ্রে যে কোণ উৎপন্ন করে তাকে রেডিয়ান বলে।

(\*)

যদি Rigid Body সম্মূর্ণ বৃত্তাকার মাথের একবার

ঘুরে আসে তখন কেন্দ্রে উৎপন্ন কোণ,

$$\theta = \frac{\text{চাপ}}{\text{ব্যাসার্ধ}} = \frac{2\pi r}{r} = 2\pi \text{ radian}$$

And in Degree,

$$2\pi = 360 \text{ degree}$$

$$\therefore 1 \text{ radian} = \frac{360^\circ}{2\pi} = 57.3^\circ \text{ (প্রায়)}$$

Angular velocity ( $\omega$ ):-

The rate of change of angular displacement with respect to time is called angular velocity ( $\omega$ )

সময়ের সাথে কৌণিক সরণের অনুপাতকে কৌণিক বেগ বলে।



If  $\Delta\theta$  angular displacement changes with time  $\Delta t$ ,

$$\text{Angular velocity, } \omega = \frac{\Delta\theta}{\Delta t}$$

Instantaneous Angular Velocity :-

The angular velocity of a body rotation at any one moment of time is called as instantaneous velocity.

$$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Angular acceleration :-

It is the time rate of change of the angular velocity and is usually designated by  $\alpha$  & expressed in radians per second.

Unit system :- SI unit  
Symbol :-  $\text{rads}^{-2}$

$$\alpha = \frac{d\omega}{dt}$$

### Ins. angular acceleration:-

It is the rate of an object rotates in a circular path at a particular moment in time.

$$\alpha = \lim_{dt \rightarrow 0} \frac{d\omega}{dt} \text{ [ms}^{-2}\text{]}$$

### Angular Momentum:- (কৌণিক ভ্রমণ)

The angular momentum of a rigid body is defined as the product of the moment of inertia & the angular velocity.

It is analogous (অনুরূপ) to linear (রৈখিক) momentum & is subject to the Fundamental Constraints of the Conservation of angular momentum principle if there is no external torque on object.



## Angular velocity

কোনো বিন্দু বা অক্ষকে কেন্দ্র করে ঘূর্ণনগত  
বেগের কন্যা ব্যাসার্ধ ভেক্টর এবং বেগবেগ  
ভেক্টর দুটাকে এক বিন্দু বা অক্ষের  
সাথে কন্যাটির বেগে বলে।

$$\text{moment of momentum} = mvr$$

$$\text{So, Angular Momentum, } L = mvr = m(\omega r)r = m r^2 \omega$$

$$L = I\omega = I \left( \frac{2\pi N}{t} \right)$$

(Angular momentum) :  $\text{kg m}^2 \text{s}^{-1}$

$$L = \text{Angular Momentum} \rightarrow \text{kg m}^2 \text{s}^{-1}$$

$$I = \text{জড়তার গুণক} \rightarrow \text{kg m}^2$$

$$\omega = \text{কৌণিক বেগ} \rightarrow \text{rad s}^{-1}$$

$$N = \text{আবর্তন সংখ্যা}$$

$$t = \text{সময় (s)}$$

$$\text{মাত্রা: } - M L^2 T^{-1}$$

$$L = \vec{r} \times \vec{p}$$

কন্যা বেগ

অবস্থান ভেক্টর

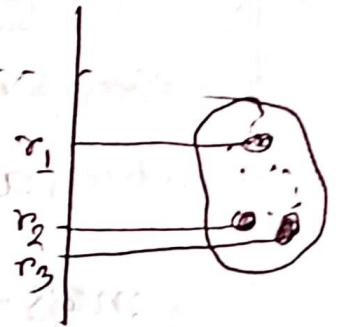
□

## Moment of Inertia (নির্মিত, মুহূর্ত) :- ( $m$ , জড়তার ভ্রামক)

The sum of product of mass & the square of perpendicular distance of each particle of an object from axis of rotation is called moment of inertia of the object.

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$

$$= \sum m_i r_i^2$$



$$\therefore I = m r^2 \quad [\text{জড়তার ভ্রামক}]$$

$\Rightarrow$  কোনো নির্দিষ্ট অক্ষের সাপেক্ষে থেকে কোনো দৃঢ় বস্তুর প্রত্যেকটি কণার ভর দূরত্বের বর্গ এবং প্রত্যেকটির ভর এর গুণফল এর সমষ্টিকে ঐ অক্ষের সাপেক্ষে ঐ বস্তুর জড়তার ভ্রামক বলে

Note:-

কোনো নির্দিষ্ট অক্ষ বরাবর একক সমকোণিক বেগে আবর্তনকৃত কোনো দৃঢ় বস্তুর জড়তার



গ্রামক, সংখ্যাতত্ত্ব এর সাতিকিঙ্ক  
 স্থিতি,  
 অর্থাৎ,

জড়তার গ্রামক,  $I = 2E$

Linear (সৈদ্বিক)	Angular (কৌণিক)
$s$	$\theta$
$v$	$\omega$
$a$	$\alpha$
mass $\rightarrow m$	$I = \text{moment of Inertia}$
$i$	$L$
Force $\leftarrow F$	$T \rightarrow \text{Torque}$

$E_k = \frac{1}{2} I \omega^2$

## Conservation of angular momentum

(কৌণিক ভ্রমণের সংরক্ষণ)

For a body rotating about an axis,

change in angular velocity can be

constant about  $\omega$  by an external

agency called ~~torque~~ torque. In the

case of a system when the external

torque is zero, the angular momentum

of the system remains constant. This is

called conservation of angular momentum

principle.  $\omega$  হলে  $L$  কৌণিক ভ্রমণের

সংরক্ষণ শর্তে Angular velocity বাকি হয়।

**Short cut** -

The law of conservation of angular momentum states that, when no

external torque acts on an object,

no change of angular momentum will occur.



## Torque

Considering (ধরি), a particle of mass  $(m)$  moving about an axis in a circular path of radius  $(r)$ . Let an external force  $(F)$  act on the particle along the tangent to the circular path.

⇒ কোনো বিন্দু বা অক্ষকে কেন্দ্র করে ঘূর্ণনমান কোনো কণার ব্যাসার্ধ থেকে এবং কণার উপর প্রযুক্ত বলের থেকে ঘূর্ণনমূলক মুহূর্তকে ঐ বিন্দু বা অক্ষের সাপেক্ষে কণাটির উপর প্রযুক্ত টর্ক বলে।

The moment of the force =  $F \cdot r$

The moment of force is called torque & represented by,  $\tau$

$$\therefore \tau = F \cdot r = m a r = m \cdot \alpha r = m \alpha r^2$$

$$\therefore \tau = I \alpha$$

Hence, Torque is equal to product of moment of inertia & angular acceleration. Torque can also be defined by the rate of change of angular momentum.

$$\begin{aligned} \tau &= \frac{dL}{dt} = \frac{d(I\omega)}{dt} \\ &= I \cdot \frac{d\omega}{dt} + \omega \cdot \frac{dI}{dt} \\ &= I \cdot \alpha + 0 \\ &= I\alpha \end{aligned}$$

$$\tau = I\alpha$$

SI unit -  $\text{kg m}^2 \text{s}^{-2}$

Other units: pound-force-foot, lbf·inch, ozf·in.

Common Symbols:  $M$

Dimension:  $ML^2T^{-2}$

SI unit :- newton metre



## Perpendicular Axes Theorem 2

(নম্ব অক্ষ উপপাদ্য)

$\Rightarrow$  The moment of inertia of a plane lamina about an axis perpendicular to the plane is equal to the sum of the moments of inertia about two mutually perpendicular axes in the plane of the lamina such that the three mutually perpendicular axes have a common point of intersection.

$\Rightarrow$  কোনো সমতল পাতের জন্য অবস্থিত দুই

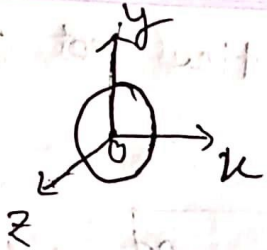
পরস্পর নম্ব অক্ষের সাপেক্ষে এ পাতের কণার

প্রাকক দ্বার সমষ্টি হবে এ দুই অক্ষের

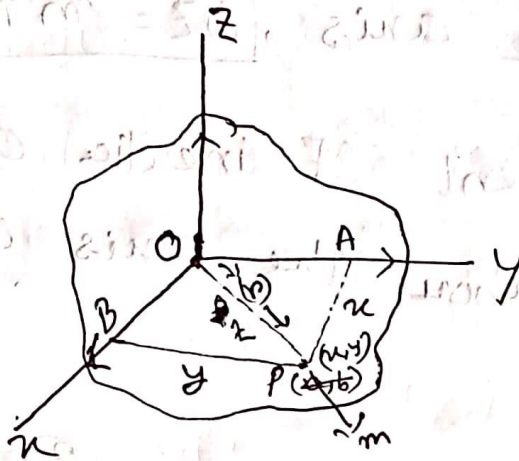
ছেদবিন্দু দিয়ে এবং পাতের অভিলম্বভাবে গমনকারী

অক্ষের সাপেক্ষে পাতটির কণার প্রাকক-এর সমান।

70. Prove that  $(I_z = I_x + I_y)$



PROOF :-



Considering a plane lamina having the axis  $Ox$  or  $Oy$  in the name of lamina. The axis  $Oz$  passes through  $O$  & is perpendicular to the plane of the lamina. Let the lamina be divided into a large number of particles, each of mass  $(m)$ . Let a particle of mass  $m$  be at  $P$  with coordinates  $(x, y)$  & situated at a distance  $r$  from the point of intersection of the axis.

$$r^2 = x^2 + y^2 \quad \text{--- (1)}$$



∴ The moment of inertia of the particle  $P$  about the axis  $OZ$  is given by,

$$I_{OZ} = m r^2 \quad \text{--- (1)}$$

The moment of inertia of the whole lamina about the axis  $OZ$  is given by,

by,

$$I_Z = \sum m r^2 \quad \text{--- (2)}$$

The moment of inertia of the whole lamina about the axis  $Ox$ ,

$$I_{Ox} = \sum M y^2 \quad \text{--- (3)}$$

Similarly,  $I_{Oy} = \sum m x^2 \quad \text{--- (4)}$

From equation (1) & (2)

$$I_Z = m (x^2 + y^2)$$

$$= m x^2 + m y^2$$

$$= I_y + I_x \quad \therefore I_Z = I_x + I_y$$

Note:-

$\sum M x^2 = I_y$  and  $\sum M y^2 = I_x$

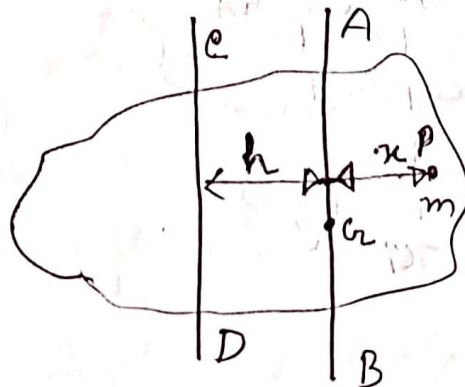
### Shortcut:-

The perpendicular axis theorem states that, the moment of inertia of a planar lamina about an axis perpendicular to the plane of the lamina is equal to the sum of the moments of inertia of lamina about the two axes at right angles to each other, in its own plane intersecting each other at the point where the perpendicular axis passes through it.

### Parallel axis Theorem :-

(समानुमान तत्र उभयम्)

The  $m$

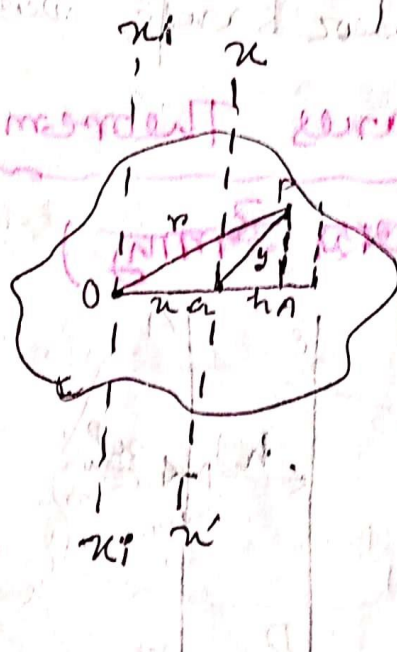


The moment of inertia of a body



about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through the center of mass & product of the mass of the body & the square of the perpendicular distance between the two parallel axes.

$$I_0 = I_G + mn^2$$



Here,

$$OG = n \quad OP = r$$

$$GP = h \quad GA = h$$

mass of P, particle is m.

$$I_0 = \sum m r^2 \quad \text{--- (1)}$$

$\Delta OPA$  - 4,

$$\begin{aligned} r^2 &= OA^2 + AP^2 \\ &= OA^2 + GA^2 \quad \text{--- (2)} \end{aligned}$$

~~किस~~ (मातृ दायक) situation to transform of  $\Delta OPA$  शक नहीं, moment of Inertia

$$GP^2 = GA^2 + AP^2$$

$$\therefore AP^2 = GP^2 - GA^2 \quad \text{--- (3)}$$

(2) नर (क) (1) नर ए समाने

$$= (x+h)^2 + y^2 - h^2$$

$$= x^2 + 2xh + h^2 + y^2 - h^2$$

$$= x^2 + 2xh + y^2 \quad \text{--- (4)}$$

(i) नर शक,

$$I_0 = \sum m r^2 = \sum m (x^2 + 2xh + y^2)$$

$$= \sum m x^2 + \sum 2m x h + \sum m y^2$$

$$= m x^2 + I_G + 0 = I_G + m h^2$$



Case,

$$\sum my^2 = I_c$$

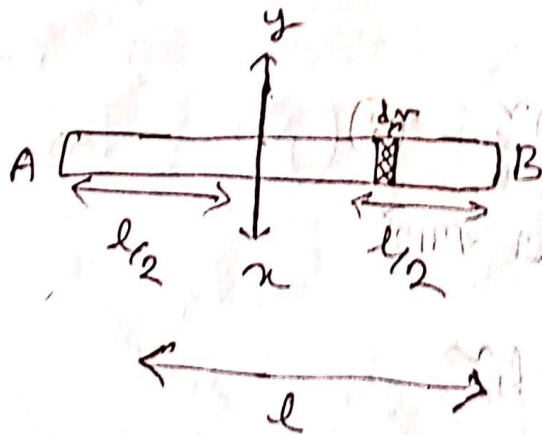
$$\sum mh = 0$$

$$\sum mgh = 0 = m \cdot r \cdot h$$

$$\sum mh = 0 \quad [g = \text{constant}]$$

□ Moment of Inertia (दुर्बल क्षमता) :-

PROOF :-



mass of bar =  $M$

Length of bar =  $l$

Mass per unit =  $\frac{M}{l}$

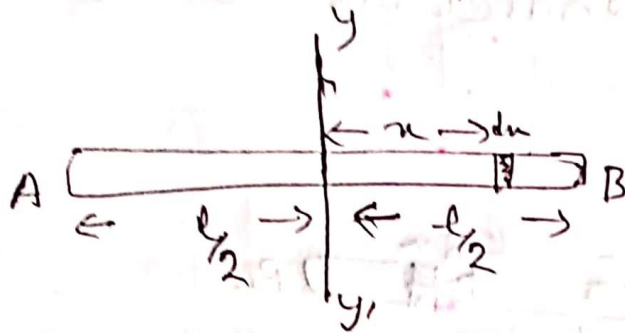
Now,

Take an element of length  $dx$  at

a distance  $x$  from the axis,

$$\text{Mass of the element} = \left(\frac{M}{l}\right) dx$$

$\Rightarrow$



Moment of inertia of the element about the axis  $yy'$

$$= \left[ \left(\frac{M}{l}\right) dx \right] x^2$$

Moment of inertia of the bar AB about the axis  $yy'$

$$= \int \left[ \left(\frac{M}{l}\right) dx \right] x^2$$

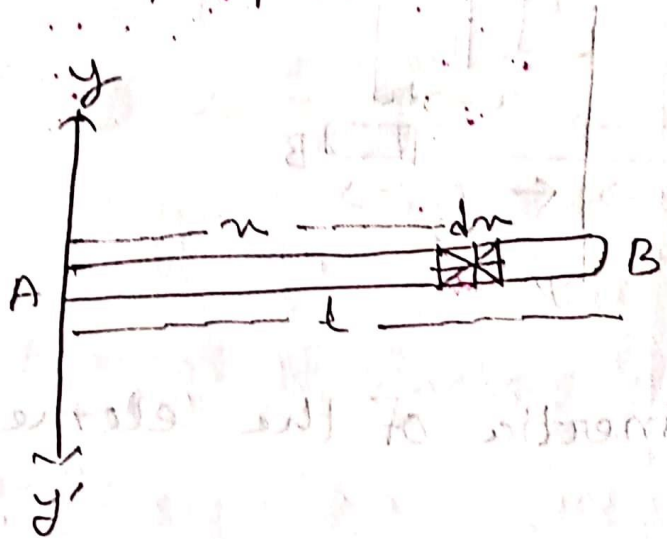
Moment of inertia of the bar AB about the axis  $yy'$

$$I = 2 \int_0^{l/2} \left(\frac{M}{l}\right) x^2 dx = \frac{2M}{l} \left[ \frac{x^3}{3} \right]_0^{l/2}$$

$$I = \frac{Ml^3}{12} = Mk^2 \text{ or, } Mk^2 = \frac{Ml^3}{12} \therefore k = \frac{l}{2\sqrt{3}}$$



□ Moment of inertia of a bar about an axis passing through one end and perpendicular to its length,



Here,

$$I = \int_0^l \left( \frac{M}{l} \right) x^2 dx$$

$$I = \frac{M}{l} \int_0^l x^2 dx \quad \text{--- (3)}$$

$$\text{or, } I = \frac{M}{l} \left[ \frac{x^3}{3} \right]_0^l$$

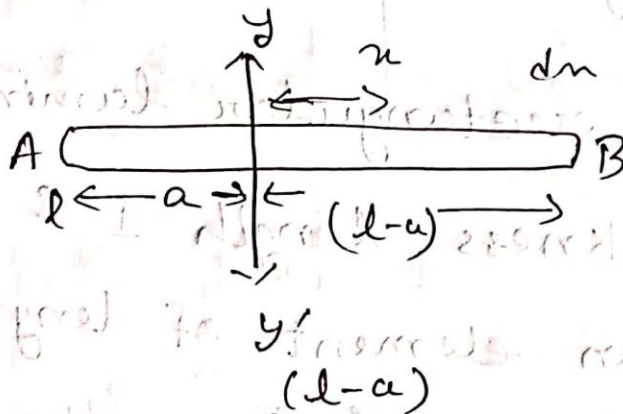
$$= \frac{M}{l} \frac{l^3}{3}$$

$$= \frac{Ml^2}{3}$$

But  $I = MK^2$  so,  $MK^2 = \frac{Ml^2}{3}$

$\therefore k = \frac{l}{\sqrt{3}}$  (4) From a rod of length  $l$

To its length, at a distance from one end:-



Here,  $I = \int_{-a}^{(l-a)} \left(\frac{M}{l}\right) x^2 dx$

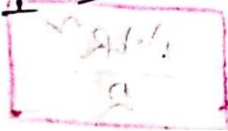
or,  $I = \frac{M}{l} \left[ \frac{x^3}{3} \right]_{-a}^{(l-a)}$

or,  $I = \int_{-a}^{(l-a)} \left(\frac{M}{l}\right) x^2 dx$

$= \frac{M}{l} \left[ \frac{x^3}{3} \right]_{-a}^{(l-a)}$

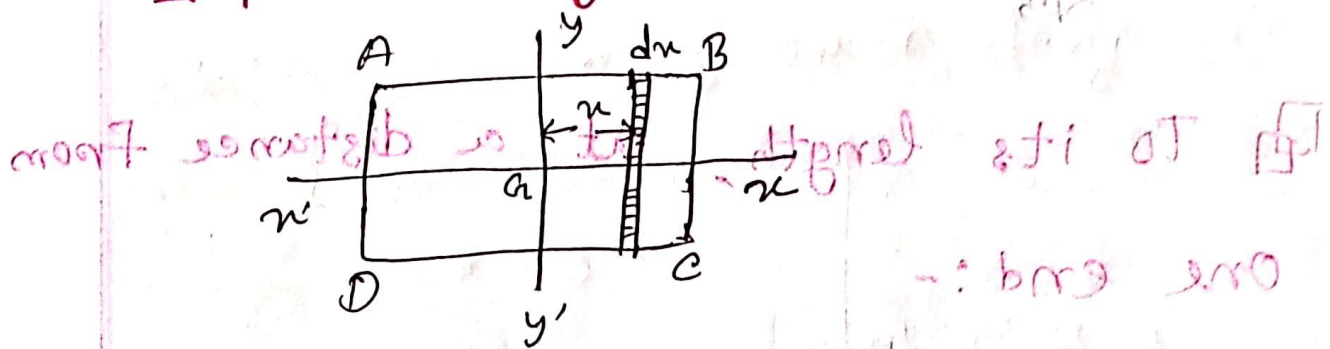
$= \frac{M}{l} \cdot \frac{1}{3} \left[ (l-a)^3 - (-a)^3 \right]$

$\therefore I = \frac{M}{l} \left[ \frac{2}{3} - la + a^2 \right]$





□ From a regular Lamina: -



⇒ Consider a rectangular lamina of uniform thickness, length  $l$  & breadth  $b$ . Consider an element of length  $dx$  at a distance  $x$  from the axis  $yy'$ .

Mass of the lamina =  $M$

Area of the lamina =  $l \times b$

Mass per unit area =  $\left( \frac{M}{l \times b} \right)$

$$\therefore I = \frac{2M}{R^2} \left[ \frac{R^4}{4} \right]$$

$$= \frac{MR^2}{2}$$

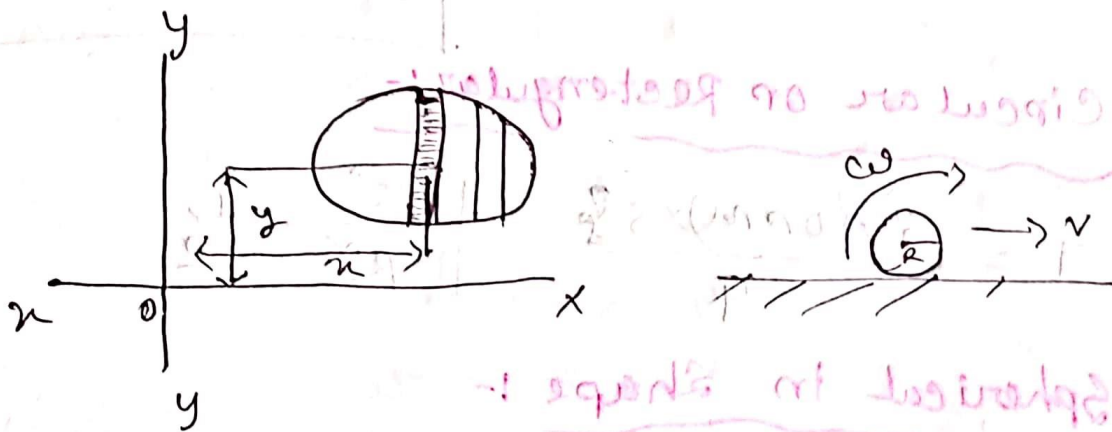
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Moment of inertia of a circular disc about its diameter

$$\boxed{\frac{MR^2}{2}}$$

## Routh's Rule:-

In classical mechanics, the stretch rule (sometimes referred to as Routh's rule) states that the moment of inertia of a rigid ~~subject~~ object is unchanged when the object is stretched parallel to an axis of rotation that is a principal axis, provided that the distribution of mass remains unchanged except in the direction parallel to the axis.



Radius of gyration

(वक्रांतर परिमाण)

$$v = \omega R ; I = MK^2$$



For Bodies :-

Square in shape or Rectangular :-

$$I = \frac{\{(A \text{ or } M) \times S\}}{3}$$

A = Area of the body

M = Mass of the body

S = Sum of the squares of the couple of semi

drawing the one about which the m.o.I being calculated.

Circular or Rectangular :-

$$I = \frac{\{(A \text{ or } M) \times S\}}{4}$$

$$\frac{K^2}{R^2} = \frac{1}{2}$$

Spherical in shape :-

$$I = \frac{\{(A \text{ or } M) \times S\}}{5}$$

$$\frac{K^2}{R^2} = \frac{2}{5}$$

□ কোনো বস্তু ঢাল (বস্তু হাড়িসে) চাড়ত থাকলে

স্থান:-



Loss  $E_p = mgh$

$\sin \theta = \frac{h}{l}$  Or,  $h = l \sin \theta$

$E_p$  (Loss of potential Energy) =  $mgh \sin \theta$

Gain in  $E_k$

=  $\frac{1}{2} m v^2 (1 + \frac{k^2}{R^2})$  — (1)

=  $mgl \sin \theta$

Or,  $v^2 = \frac{2gl \sin \theta}{1 + \frac{k^2}{R^2}}$  — (2)

Derivative of  $v$  to  $t$  of equation (2)

$2v \frac{dv}{dt} = \frac{2g \sin \theta}{(1 + \frac{k^2}{R^2})} \cdot \frac{dl}{dt}$

$\therefore a = \frac{g \sin \theta}{1 + \frac{k^2}{R^2}}$  — (3)



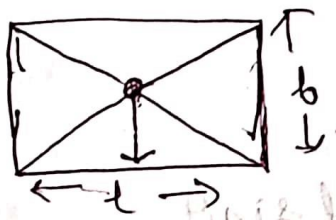
Semi Square

$\angle \rightarrow$  one-eight of  $360^\circ$  circle

For,  $a = \frac{2gs \sin \theta}{3}$

$$= \frac{7gs \sin \theta}{5}$$

$\square$  Routh's rule (घातना ७ भूखण्ड आसुतकण्ड घात)



$$I = I_x + I_y$$

$$= \frac{mb^2}{12} + \frac{ml^2}{12}$$

$$I = \frac{m}{3} \left\{ \left( \frac{l}{2} \right)^2 + \left( \frac{b}{2} \right)^2 \right\}$$

$$= \frac{m}{3} \frac{l^2 + b^2}{4} = \frac{m}{12} (l^2 + b^2)$$

$\frac{1}{n} (n)$  [sum of other two semi squares]

where,

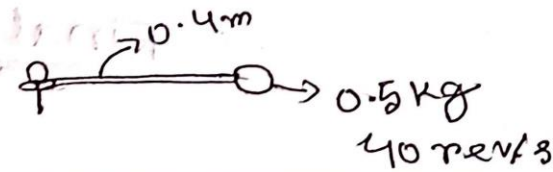
$n=3$  For rectangular body.

$n=4$  For circular/elliptical body.

$n=5$  For spherical body.

Rule - If a body is symmetrically shaped about all its three axes, then the moment of inertia about any three axes which passes

Through the center of gravity of the body can be written  
as,  $I = \frac{MR^2}{5}$  - spherical shape



Q A 0.5 kg mass is whirled in a circle at the end of a string 0.4 m long, the other end of which is held in the hand. If the mass makes 40 revolutions per second, what is ~~this~~ its angular momentum.  
If the the number of revolution decreases by one in 20s., Calculate the mean value of the torque on the system.

$\Rightarrow$



Date:- August 15, 20

Saturday

11:25 p

Elasticity (प्रत्याहारिता)

(प्रतिबलप्रतिफल)

Elasticity :-

Elasticity is defined as the property by which a body regains its original position when the forces are withdrawn.

The opposite of elasticity is plasticity. No substance is perfectly elastic or perfectly plastic.

Stress (प्रित्त) :-

When a force is applied on a body, there will be relative displacement of the particles and due to the property of elasticity the particles tend to regain their original position.

Stress is defined as the restoring force per unit area.

## Normal Stress (लम्बवर्धन तनाव) :-

Or Longitudinal Stress :-

Restoring force per ~~area~~ unit area perpendicular to the surface is called

Normal / Longitudinal Stress.

## Tangential Stress :- (अनुप्रस्थ / स्पर्शक तनाव) :-

Restoring force parallel to the surface per unit area is called tangential stress. (Shearing  $\rightarrow$ )



## Strain :- (वर्धन) / (विकृति)

The ratio (वर्धन) of the change in shape to the original shape is called

strain. There are three types of

strains:-



## Longitudinal / Lateral Strain (নমুনাটির বিকৃতি) (লম্বা বা প্রস্থ)

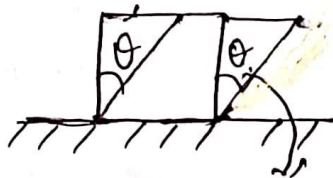
The ratio of change in length to original length is called longitudinal strain.

$$\left(\frac{\Delta l}{L}\right)$$

Shearing Strain (বিকৃতি) Actually, It is defined as the tangent of that angle.

Shear strain is the ratio of the change in deformation (বিকৃতি) to its original length perpendicular to the axes of the member due to shear stress. Shear stress is stress in parallel to the cross section of the structural member.

এটা দিলে  
কোনো  
সামান্য  
মাত্রায়



It has no  
any unit

Shearing strain.

Angle of shear measured in radians

Volume strain :- (आमूलन विकृति) (विकृति-सिद्धान्त)

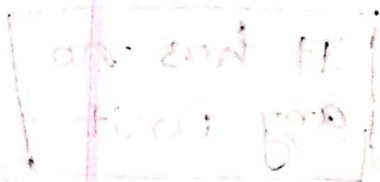
The ratio of the change in volume to original volume is called volume strain

$$\text{Formula :- } \left( \frac{\Delta V}{V} \right)$$

Elastic Limit :-

The maximum stress which a body exhibits the property of elasticity is called elastic limit.

If the applied force exceeds the maximum stress limit, the body does not regain its original position completely after the external forces are withdrawn.



Angle of shear measured in radians



Hooke's Law:-

It states that within the elastic limit, Stress is directly proportional to strain.

(-मिडन & विकृति)

Stress & Strain

Stress = E \* strain [E = constant]

E =  $\frac{\text{Stress}}{\text{Strain}}$  Modulus of elasticity

Or,  $\frac{F/A}{l/L} = y$  [Young's Modulus of elasticity]

Or,  $\frac{PL}{Al} = y$  [Modulus of Rigidity]

$\eta = \frac{F/A}{\theta} = \frac{F}{A\theta}$

[Normal Stress]  $\leftarrow F/A$  [Shearing strain]  $\rightarrow \theta$

$K = \frac{F/A}{V/V} = \frac{FV}{AV}$  [Bulk Modulus of elasticity]

$\leftarrow F/A$  [Normal Stress]  $\rightarrow V/V$  [Volume Strain]

$p = 0$

Poisson's ratio  
~~Poisson's ratio~~

## Poisson's Ratio :-

Whenever a body is subjected to a force in a particular direction, there is a change in dimensions of the body in the other two perpendicular directions. This is called lateral strain.

⇒ [যখন কোনো বস্তুর উপর বল প্রয়োগ করা হয় তখন কোনো দিকে তখন, এর আকারে পারিষ্কার হয়ে যায় একই বস্তু উল্লম্ব বিকৃতি]

Lateral strain is proportional to the size of the body.

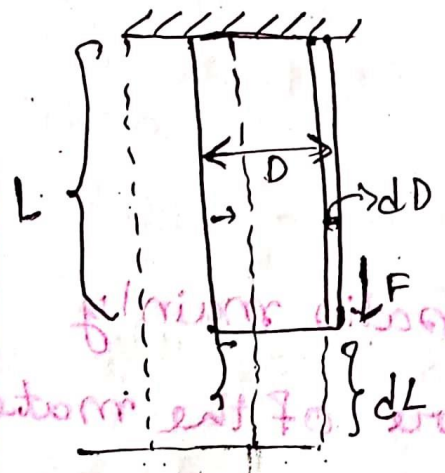
Let the  $(\alpha)$  be the longitudinal strain per unit stress and  $\beta$  be the lateral strain per unit stress, within the elastic limit.

$$\beta = \sigma \alpha$$

So, Poisson's ratio,  $\sigma = \frac{\beta}{\alpha}$



\* Lateral strain per unit to the longitudinal strain ~~per~~ per unit stress is called poisson's ratio.



$$\alpha = \frac{dL}{L} \quad \text{[दिर्घ विकृति]}$$

$$\beta = \frac{dD}{D} \quad \text{[संक्षार विकृति]}$$

So,  $\beta = \frac{dD/dD}{dL/L}$

$$\therefore \sigma = \left( \frac{dD}{dL} \right) \left( \frac{L}{D} \right)$$

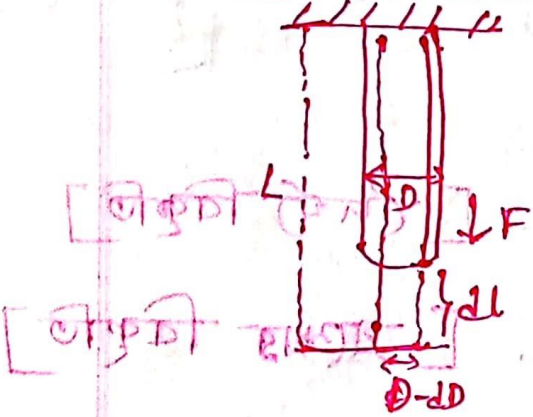
The values of  $\sigma$  will varies from 0.2 to 0.4

Initial value :- (प्रारम्भिक आयतन) <sup>volume</sup>

... (faint handwritten notes and scribbles)

Initial value of the wire is  $\sigma_0$  (1)

When a force is applied, the wire is stretched and its length increases.



poisson's ratio

(\*) The value of poisson's ratio mainly depends upon the nature of the material of the body.  $\sigma$  has no units as it is a ratio of two numbers.

For most of the substances, the value of  $\sigma$  varies between 0.2 to 0.4.

$\Rightarrow$  If the volume of the wire remained unchanged after the force has been applied, then,

Initial volume of the wire,  $v = \pi r^2 L$

$$v = \left( \frac{\pi D^2}{4} \right) L \quad \text{--- (ii)}$$



Differentiating equation (ii)

$$dv = \frac{\pi}{4} [D^2 dL + 2LD dD]$$

If,  $dv = 0$

Then,

$$D^2 dL + 2LD dD = 0$$

$$\frac{dD}{dL} \times \frac{L}{D} = -1/2$$

But,

$$\rho = - \left( \frac{dD}{dL} \times \frac{L}{D} \right)$$

$$= -(-1/2)$$

$$= 1/2 \rightarrow$$

This is the maximum possible value of poisson ratio

$$\rho = - \left( \frac{dD}{dL} \right) \left( \frac{L}{D} \right)$$

## Alternative Method

☐ The relation between  $\gamma$ ,  $\eta$  &  $K$  :-

The relation between  $\gamma$ ,  $\eta$  and  $K$  can also be obtained with the help of the following table.

Consider a unit cube, which is subjected to outward <sup>(बाह्य)</sup> elongational <sup>(विस्तार)</sup> force  $[P]$  on each face. Let  $(\sigma)$  be the poisson's ratio for the material. In the table, the values of applied stress & the corresponding <sup>(अवस्था)</sup> strains <sup>(विकृति)</sup> produced along the three perpendicular axes are shown. For a stress  $(P)$  the longitudinal strain produced =  $\frac{P}{Y}$  in its own direction & the corresponding strains in the other two perpendicular directions are  $-\frac{\sigma P}{Y}$  &  $-\frac{\sigma P}{Y}$



Stress: (Along  $x$ ) : Strain along

$\sigma_x$	$\sigma_y$	$\sigma_z$	$\sigma_x$	$\sigma_y$	$\sigma_z$
$+P$	$0$	$0$	$+\frac{P}{y}$	$-\frac{2P}{y}$	$-\frac{2P}{y}$
$0$	$+P$	$0$	$-\frac{2P}{y}$	$+\frac{P}{y}$	$-\frac{2P}{y}$
$0$	$0$	$+P$	$-\frac{2P}{y}$	$-\frac{2P}{y}$	$+\frac{P}{y}$

volume

$+P$	$+P$	$+P$	$\frac{P}{y}(1-2\epsilon)$	$\frac{P}{y}(1-2\epsilon)$	$\frac{P}{y}(1-2\epsilon)$
Total volume strain :-			$\frac{3P}{y}(1-2\epsilon)$		

Shearing

$+P$	$0$	$0$	$+\frac{P}{y}$	$-\frac{6P}{y}$	$-\frac{6P}{y}$
$0$	$-P$	$0$	$+\frac{6P}{y}$	$-\frac{P}{y}$	$+\frac{6P}{y}$
Sum:-	$-P$	$0$	$\frac{P}{y}(1+6)$	$-\frac{P}{y}(1+6)$	$0$

Total

Shearing strain:  $-\frac{2P}{y}(1+6)$

## Bulk Modulus: (আম্লতা সূচক):-

Bulk modulus is used to measure how incompressible (অসংকোচনীয়) a solid is.

Besides, the more the value of  $K$  (Indicator of Bulk modulus) for a material, the higher is its value nature to be

incompressible. For example, the value of  $K$  for steel is  $1.6 \times 10^{11} \text{ Nm}^{-2}$  & the value of  $K$  for glass is  $4 \times 10^{10} \text{ Nm}^{-2}$

It is defined as the ratio of the infinitesimal (সূক্ষ্মতরঙ্গ) pressure increase to the resulting relative decrease of the volume.

Its unit is the = Pascal.

$$\text{Formula is } = \frac{\Delta V}{V}$$



## Shearing Strain (Deformation):

tangent of angle and is equal to the length of deformation at its maximum divided by the perpendicular length in the plane of force.

Now,

$$\text{B.M.}, k = \frac{P}{\text{Strain}}$$

$$\text{So, Strain} = \frac{P}{k}$$

$$\text{or, } \frac{3P}{y} (1-2\epsilon) = \frac{P}{k}$$

$$\text{or, } k = \frac{Py}{3P(1-2\epsilon)}$$

$$\text{or, } k = \frac{y}{3(1-2\epsilon)} \quad \text{--- (i)}$$

Now,

$$\text{Shearing strain} = \frac{P}{\eta} \quad [\text{we}]$$

From the table,

$$\text{Shearing strain} = \frac{2P}{y} (1+\epsilon)$$

$$\text{So, } \frac{P}{\eta} = \frac{2P(1+\epsilon)}{y}$$

$$\text{or, } \eta = \frac{y}{2(1+\epsilon)} \quad [\text{divided by } P]$$

--- (ii)

So, From (1) & (2),

$$1 - 2\theta = \frac{y}{3R}$$

$$1 - 2\theta = \frac{y}{3K} \quad \text{--- (iii)}$$

$$2 + 2\theta = \frac{y}{\eta} \quad \text{--- (iv)}$$

---


$$3 = \frac{y}{3K} + \frac{y}{\eta}$$

$$\text{Or, } 3 = y \left( \frac{1}{3K} + \frac{1}{\eta} \right)$$

$$\therefore \frac{3}{y} = \frac{1}{3K} + \frac{1}{\eta} \quad \text{--- (5)}$$

Limiting values of  $\theta$  :-  $[-1 < \theta < 0.5]$

$$\eta = \frac{1}{2(1+\theta)} = \frac{1}{2(1+\frac{\beta}{2})}$$

$$\eta = \frac{y}{2(1+\theta)} \quad \text{--- (i)}$$

From equation (iii) & (iv)

$$y = 3K(1 - 2\theta) \quad \text{--- (3) (1)}$$

$$y = 2\eta(1 + \theta) \quad \text{--- (4) (2)}$$



$$\therefore 3k(1-2\delta) = 2n(1+\delta)$$

When,

$\delta = +$  (positive) then,  $k$  &  $n =$  positive (+)

Limit: IF the Poisson's ratio is a positive quantity

# Limit: - as  $k$  &  $n$  are always positive

$$1-2\delta > 0$$

$$\text{OR, } 2\delta < 1$$

$$\text{So, } -1 < \delta < 0.5 \leftarrow \text{Limit}$$

$\Rightarrow$  IF the

(b) IF the Poisson's ratio is a negative quantity, for  $n$  &  $k$  to be positive,

$$(1+\delta) > 0$$

$$\delta > -1$$

It means the value of  $\delta$  lies between,

Note:-

$\Rightarrow$  In actual practice the value of  $\delta$

cannot be negative because the body doesn't expand laterally when it

expands longitudinally (लम्बान्निवृत्तता). And

when  $\sigma = 0.5$ , it means that there is no

(\*) change in volume and the body is

completely incompressible. Therefore, it is not possible.

In practice, the value of ( $\sigma$ ) for most

of the isotropic ~~sol~~ substances (समसाम्यता)

is between 0.2 & 0.4.

$\mu$  = the modulus of rigidity (समसाम्यता)

$\sigma$  = poisson's ratio.

$Y$  = young's modulus.

$\square$  yield point (समत विन्दु) :-

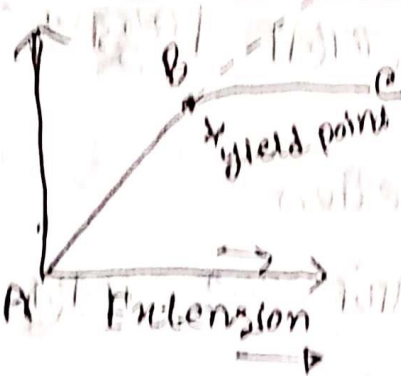
where a wire <sup>(spring)</sup> is loaded beyond elastic

limit, Hooke's law is no longer obeyed.

The extension is more than the corresponding load.

In the above graph B point is yield point.





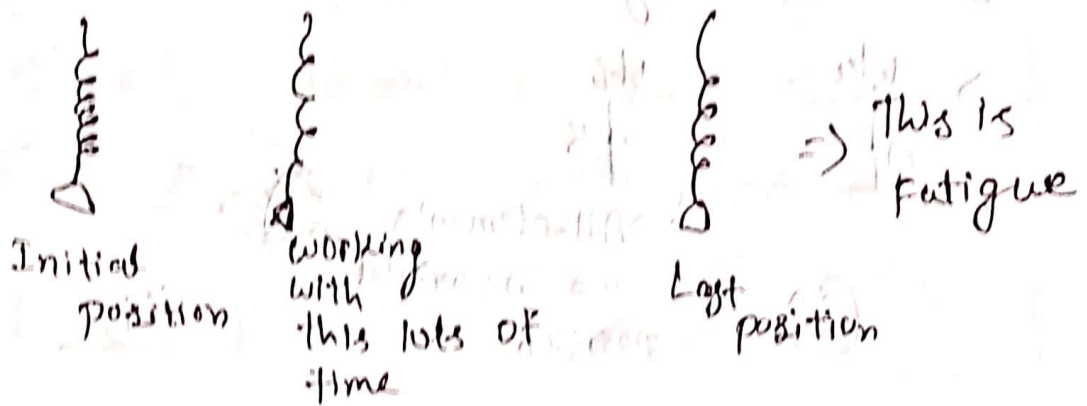
If the load is removed, the particle don't regain their position

Note:-

এই গ্রাফে একটি পয়েন্টে  
ইকম নীতি মাত্র চলবে না

IV Elastic Fatigue:-

IF a body is continuously subjected to stress & strain it gets fatigued.



V Bending of Beams:-

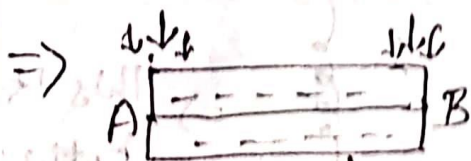
A beam is defined as a structure of uniform cross-section, whose length is large comparison to its breadth & thickness.

(এটি সমতল/অনু ক্রান্তে কিছুই হতে পারে)

Uniform cross-section

[কোথাও সরু, মোটা হতে পারবে না। একদম সমান হতে হবে]

□ Consider a beam of uniform rectangular cross section. Let the beam be subjected to deforming forces so that it bends.



(i)

All elements are in parallel position.



(ii)

Position No. (i) :-

Let, AB beam be subjected to deforming forces, so that it bends. In the initial position of the beam, the various filaments constituting the beam are in parallel

layers of equal length. বিস্তারিতভাবে বিস্তারিতভাবে  
সমানভাবে বিস্তারিত।



position No. (ii):-

In the final position, above the layer AB the filaments are elongated (কৃৎ শক্তি মাওয়া) while below AB they are compressed (সংকুচিত)

The length of the layer AB remains an altered. It is called the neutral axis.

The surface combining the neutral axis is called neutral surface.

Mid Term Finishes

Final Starts From

Next Page



**KEEP  
CALM  
ITS TIME FOR THE  
FINAL  
EXAM**



# Final

## Lesson: - Surface Tension

### Cohesive Adhesive Force:-

Mutual attraction between the molecules of the same substances is called cohesion (असंलग्नता).

And the force is called as cohesive force.

### Adhesive Force:-

Mutual attraction between the molecules of different substances is called adhesion (संलग्नता लेना शक्ति).

And the force is called as Adhesive force.

### Surface Tension:-

(\*) These forces are effective only when the distance between the neighbouring molecules is extremely small of the order  $10^{-7}$  cm.

- The greatest distance at which the molecules can attract each other is called molecular

range.

- A sphere having a radius equal to the molecular range, with the molecule as the centre is called the sphere of influence. (संभावित क्षेत्र)

### Surface Tension

It is the property of a liquid by which the free surface of a liquid behaves like a stretched membrane.



⇒ Consider an imaginary line AB is dividing the liquid surface the molecules of the liquid in to two parts.

The molecules on one side pull away the molecules on the other



side. Therefore, if the liquid was actually separated into two parts with AB as the boundary line, work would have to be done.

(more explanation)  
production of a large amount of a free surface requires an increase in the potential energy of the liquid. The undisturbed liquid tries to minimize its potential energy. So, it must take a shape in which its free surface area is as small as possible.

Its surface tends to contract and force per unit length acting on either side of the imaginary line drawn on the liquid surface at rest is called as the surface tension.

$$\text{Surface Tension} = \frac{\text{Force}}{\text{Length}}$$

$$\therefore T = \frac{F}{L}$$

- The unit of surface tension in CGS system is  $\text{dyne per centimetre}$

And in nationalised (सोझिकी ँणव)

- MKS system is newtons per metre.

- The dimensions of surface tension are

$$[MT^{-2}]$$

7] Surface Tension Dependency:-

(i) Temperature (K):-  $T \propto \frac{1}{K}$

(ii) Density (P) :-  $T \propto P$

(iii) Impurity

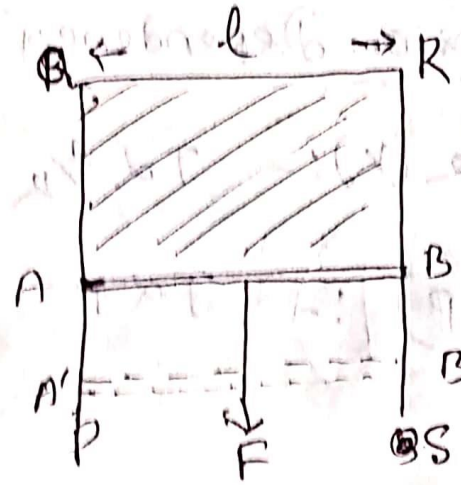


# Surface Energy & Surface Tension

## Surface Tension

As discussed already, a liquid surface contracts spontaneously. Free energy is associated with it and work must be done to extend the surface.

Let consider a rectangular frame PQRS



with horizontal wire AB. Dip it in a soap solution. A film is formed across the surface tension ABRO. The wire AB is pulled up due to surface tension of the film. To keep the wire

to equilibrium (सुस्थिति) and in its position, a force has to be applied downwards. Let the total force be  $F$ .

So,  $F$  is equal to  $w_1 + w_2$

$w_1 =$  weight of the wire

$w_2 =$  Extra weight to be used

$F$  will hold the wire at rest in any position,

regardless (निर्दिष्ट) of the area of the surface.

When the wire,  $AB$  is pulled downwards

through a distance  $\delta x$ , the area of the film

is increased. The molecules in the bulk

of the liquid move into the surface layers.

- It is assumed that the temperature remains constant

$$\boxed{\text{work done} = F \times \delta x} \quad \text{So, } w = F \cdot \delta x$$



• Total increase in the surface area of the liquid film

$$= 2(l \times \Delta x)$$

where  $l$  is the length of the wire AB

∴ work done per unit area =  $\frac{F \cdot \Delta x}{2l \cdot \Delta x} = \frac{F}{2l}$

⊗ The work done per unit area gives the increase in potential energy per unit area of the film.

⊗ The increase in energy per unit area is called surface energy.

Also the force,  $F = T \times 2l$

Here,  $T$  is the surface tension.

$$\therefore T = \frac{F}{2l}$$

⊠ It seemed that the temperature remains const. throughout the process. Actually the film gets cooled when it is stretched.

Therefore extra energy has to be supplied to restore the original temperature of the

Film.

~~S = T + H~~  $S = T + H$  (i) [ H = The quantity of heat absorbed per unit area from atmosphere ]

Or,  $T = S - H$  (ii) [ (S - H) = The mechanical part of the surface eng. / and S = The amount of work done in increasing the surface area of the film by one unit ]

If the process is

Adiabatic (अ绝热),  $H = 0$

From eq<sup>n</sup> (ii)

$$T = S$$

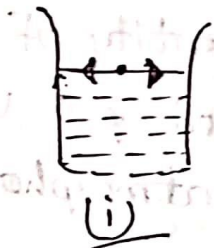
The units of free surface energy in CGS system is ergs per sq cm

In MKS system: - Joules per sq. metre.



## Pressure Difference Across a Spherical Surface :-

(i) When the liquid surface is plane, the resultant force of surface tension on a molecule is zero.



(ii) When the liquid surface is convex (outward), the resultant force is outward.



(iii) When the liquid surface is concave (inward), the resultant force is inward.



Thus for curved liquid surface the pressure on concave side is greater than on the convex side.

The difference of pressure depends upon the surface tension of the liquid and the radius of curvature of its surface.

Example :- A liquid drop of radius  $R$  breaks up into 64 small drops, calculate the

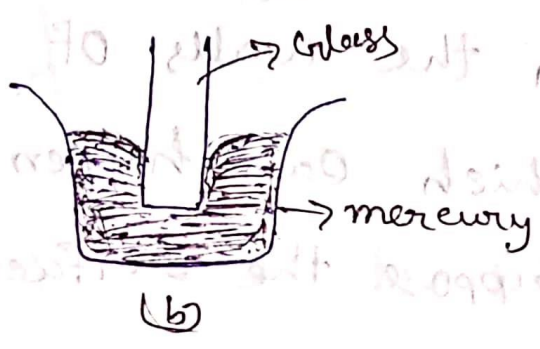
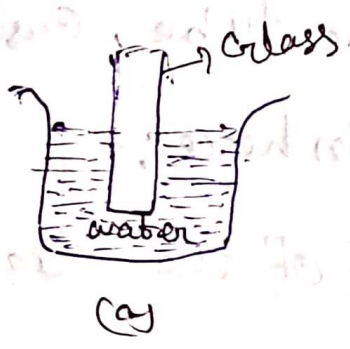
Main Formula =

Change in energy / Energy required = (Surface Tension)  $\times$  (Increase in Area)

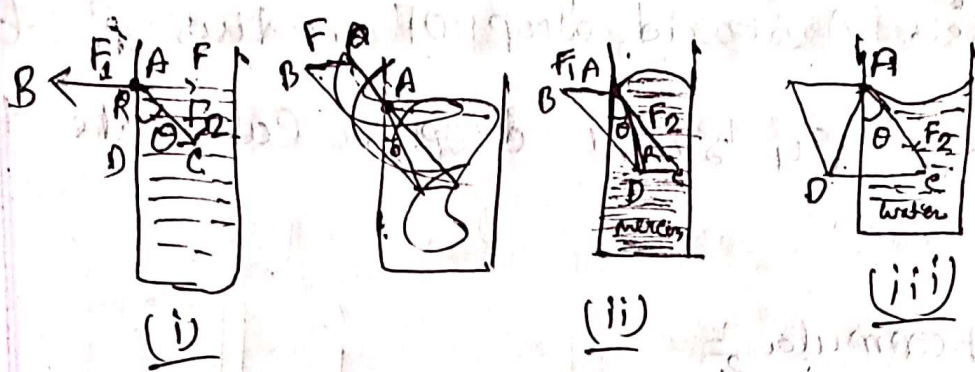
(कठि मुक्त गति शत)

Angle of Contact

संश्लेषण कोण







$\Rightarrow$  Considering a liquid in a glass tube.

The molecule, at A experiences,

(i) Force of adhesion (अनुचिपकण)  $F_1$  between it (A) & the glass molecules.

(ii) Force of cohesion (अनुचिपकण)  $F_2$  between it and the molecules of the liquid.

#  $F_1$  acts along AC making an angle of  $45^\circ$  with the walls of the tube, causes

which can happen in here:-

1) Suppose the surface of the liquid is horizontal as shown in (i)  $F_2$  is the force of cohesion acting along AC.

The component of  $F_2$  along AE is  $F_2 \sin \theta$ .

$F_1$  is the force of adhesion along AB.

$$F_1 = F_2 \sin \theta$$

Thus the molecule A does not experience any force in the horizontal direction.

The only force is in the vertical direction and is equal to  $F_2 \cos \theta$

where,  $\theta = 45^\circ$

And the angle of contact is  $90^\circ$

$$P r v_r = P_a v_a + P_b v_b \text{ or, } P r \times \frac{4}{3} \pi r^2 = P_a \times \frac{4}{3} \pi a^2 + P_b \times \frac{4}{3} \pi b^2$$

$$\text{or, } P r v_r = P_a a^2 + P_b b^2 \quad \text{Capillarity (पिण्डकण्ठ)}$$

When a capillary tube (पिण्डकण्ठ नलिका) of fine bore (सूक्ष्म पिण्ड) is dipped (डुबाना) in water, water rises in the tube.

A tube with a fine bore is called a capillary tube.

$$\therefore T = \frac{P(r v_r - a^2 - b^2)}{4(a^2 + b^2 - r v_r)}$$

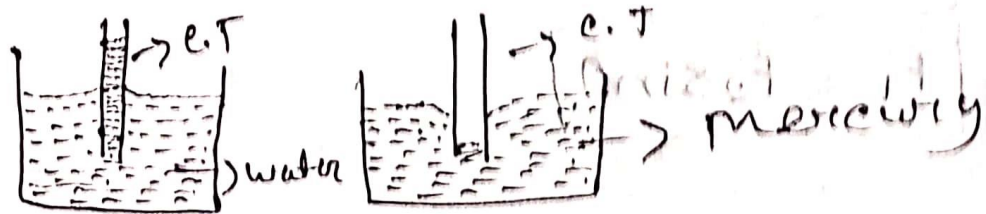
$$F_p = \frac{4T}{r} + P$$

$$P_a = P + \frac{4T}{a}$$

$$P_b = P + \frac{4T}{b}$$



On the other hand, if the same tube is dipped in mercury, there is depression of mercury level in the tube.



The property of rise or depression of a liquid inside a capillary tube is called Capillarity. It is one of the most important effects of surface tension.

In general, the liquids that wet the glass, rise inside the capillary tube while those which do not wet the glass show a depression inside the capillary tube.

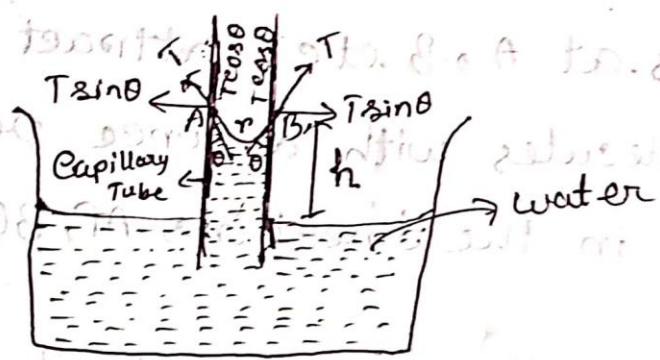
Expression for surface tension:

[To be continued]

# Surface Tension

২২.০৬.১৯ (৭৯)

## Expression for surface Tension (উল্লেক্ষ প্রমাণের ব্যাখ্যা)



⇒ Consider a capillary tube of radius  $r$  dipped in water. Due to surface tension, the water rises inside the capillary tube.

$h$  = height of water

The meniscus is concave. (point A, B)

$T$  acts tangential to the surface with the wall of the capillary tube.

The force per unit length,  $T$  refers to the



force experienced by the molecule at A.

due to,

1) The force of adhesion due to glass molecules.

2) The force of cohesion due to liquid molecules at A, B etc.

attract the liquid molecules with a force per unit length  $T$  in the directions AP, BQ etc.

$\Rightarrow$  ~~To~~ resolve the force per unit length  $T$  along AB into two rectangular components.

(i)  $T \cos \theta$  acting vertically upwards.

(ii)  $T \sin \theta$  acting in the horizontal direction.

\*

$$\text{Force per Unit length} = T \cos \theta$$

$$\text{Circumference} = 2\pi r$$

$$\text{Total upward force} = T \cos \theta \times 2\pi r$$

This upward force balances the weight of the liquid column in the capillary tube.

$$\text{Height of the liquid column} = h$$

Volume of the liquid in the meniscus (ঈর্ষাকোণ)

$$= \left( \pi r^2 \times r - \right.$$

$$\left. = \frac{\pi r^3}{3} \right)$$

Volume of the liquid in the column

$$= \pi r^2 h + \frac{\pi r^3}{3} = \pi r^2 \left( h + \frac{r}{3} \right)$$

Density of the liquid =  $\rho$

$$\text{Weight of the liquid} = \pi r^2 \left( h + \frac{r}{3} \right) \rho \cdot g$$

Equating (i) & (ii)

$$2\pi r T \cos \theta = \pi r^2 \left( h + \frac{r}{3} \right) \rho \cdot g$$



$$\cong 40\pi, T = \frac{r(h + \frac{r}{3})\rho \cdot g}{2 \cos \theta}$$

When the angle of contact is zero,

$$\cos \theta = 1$$

$$\text{Or, } T = \frac{r(h + \frac{r}{3})\rho \cdot g}{2}$$

### Determination of surface Tension:- (जल का तनाव निर्धारण)

Surface tension of water can be determined experimentally using capillary method.

The height of the liquid column & the internal diameter of the capillary tube is found with the help of a travelling microscope.

For a particular tube, the radius is  $r$ , the height of water is  $h$  and the density of water is  $\rho$ . Then the surface tension of water at room temperature

$$T = \frac{r(h + \frac{r}{3})\rho g}{2\cos\theta}$$

Taking,  $\theta = 0$ ,  $\cos\theta = 1$

$$T = \frac{r(h + \frac{r}{3})\rho g}{2}$$

Surface tension of water can be determined...

In the actual experiment, three capillary tubes of different diameters are taken and the mean value of surface tension is calculated.

This method suffers from a number of drawbacks.

1) The angle of contact has been taken to be zero, because it can't be measured inside the tube.

2) The tube may not be of uniform area of cross-section.



(iii) The tube may not be cleaned. It is clogged.

Example of Capillarity:

(1) A pen nib is split at the tip to provide the narrow capillary and the ink is drawn up to the tip

continuously.

(2) In oil lamps & stoves, oil is drawn up through the capillaries of the wick.

(3) Pores in the blotting paper act as capillaries. When the blotting paper is kept in contact with ink, the ink is absorbed.

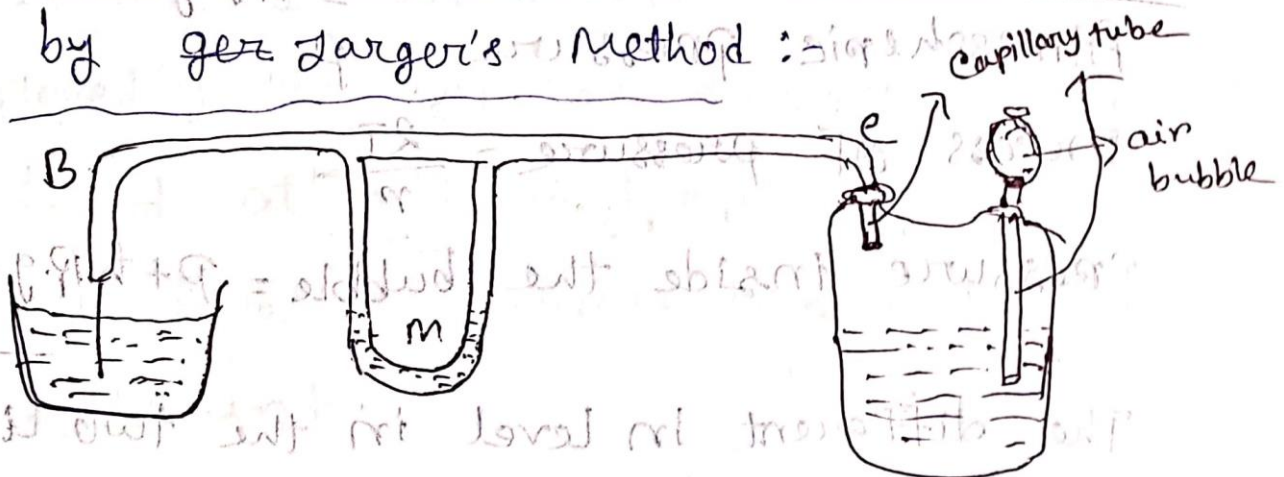
(4) Walls get damped in rainy season due to the absorption of water

by the bricks by capillary action.

5] Leaves, trunk & branches of a tree possess fine capillaries. Sap & water rises even up to the top most leaves by capillary action.

6] Sandy soils are dry whereas clay soils are damp. The interspaces between the particles of the clay form finer capillaries and water rises to the surface quickly.

7] Determine surface tension of a liquid by Jeger's method:



The apparatus (A)

D is Woulf's bottle; on one side it is fitted



with a thistle funnel and other side it is connected to a tube BC. The tube BC is fitted with a manometer. The end of the tube BC is joined to a capillary tube. Two capillary tubes one is from C & the other is inside the liquid which surface tension is to be determined.

For stop cock is opened the pressure of bottle increases. Here,

Excess of pressure inside the bubble,

$$= \frac{2T}{r}$$

The pressure of the liquid column

$$= h \rho g$$

Atmospheric pressure =  $p$

Excess of pressure =  $\frac{2T}{r}$

pressure inside the bubble =  $p + h \rho g + \frac{2T}{r}$  (1)

The different in level in the two limbs of the manometer, when the bubble just bursts =  $h$

Density of the liquid in the manometer

$$= \rho_2$$

pressure of air in the manometer,

$$P_2 = P + h\rho_2 g \quad \text{--- (ii)}$$

Equating (i) and (ii)

$$P + h\rho_1 g + \frac{2T}{r} = P + h\rho_2 g$$

$$\frac{2T}{r} = (h\rho_2 - h\rho_1)g$$

$$T = \frac{g\rho}{2} (h\rho_2 - h\rho_1)$$

The radius ( $r$ ) of the capillary tube is measured with a travelling microscope

This method can be usefully

employed (i) to find the surface tension of a liquid at various temperatures.

(ii) to find the surface tension of molten metal and

(iii) to compare the surface tension of two liquids at the same temperature.



math problems solve

Q] A liquid drop of radius (R) breaks up into 64 small drops. Calculate the change in Energy.

⇒ Let,

the surface tension of the liquid be T.

Radius of the bigger drop = R

Surface area of the bigger drop =  $4\pi R^2$

Volume of the bigger drop =  $\frac{4}{3}\pi R^3$

Let, the radius of each small drop be, r

$$\frac{4}{3}\pi R^3 = 64 \times \frac{4}{3}\pi r^3$$

$$R^3 = 64r^3$$

$$\therefore R = 4r$$

$$r = \frac{R}{4}$$

$$r = \frac{R}{4}$$

Surface area of each small drop, (1)

$$= 4\pi r^2$$

$$= 4\pi \left(\frac{R}{8}\right)^2 = 4\pi \frac{R^2}{64} = \frac{\pi R^2}{16}$$

Surface area of 64 small drops,

$$= \frac{64 \times \pi R^2}{16} = 4\pi R^2$$

Increase in surface area =

$$16\pi R^2 - 4\pi R^2$$

$$= 12\pi R^2$$

Change in energy = (Surface tension)  $\times$  (Surface Area)

$$= T \times 12\pi R^2$$

$$= 12\pi R^2 T$$

$$\frac{12\pi R^2 T}{\pi R^2} = 12T$$

$$\frac{12\pi R^2 T}{\pi R^2} = 12T$$



(2) Calculate the amount of energy needed to break a drop of petrol of volume  $10^{-6} \text{ m}^3$  into a thousand million drops of equal size. Surface tension of petrol is  $26 \times 10^{-3} \text{ Nm}^{-1}$ .

$\Rightarrow$  Let, the radius of petrol's drop be =  $R$ .

The volume of a drop of petrol =  $10^{-6} \text{ m}^3$

The surface tension of petrol,  $T = 26 \times 10^{-3} \text{ Nm}^{-1}$

We will convert it into thousand

million drops =  $10^9$

What amount of energy needed = ?

Here,

$$\frac{4}{3} \pi R^3 = 10^{-6}$$

$$\text{OR } R^3 = \frac{3 \times 10^{-6}}{4 \pi}$$

$$\text{OR, } R = \left( \frac{3}{4 \pi} \right)^{1/3} \times 10^{-2} \text{ m.}$$

$$= 1.465 \times 10^{-3}$$

~~If let,  $r$~~  ~~radius of small drop~~

Also,

The radius of small drops  $= r$

So, volume of  $10^9$ 's drop of petrol

$$be = \frac{4}{3} \pi r^3 \times 10^9$$



Here,

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r^3 \times 10^9$$

$$\text{or, } \cancel{1.32 \times 10^8} = \cancel{\frac{4}{3}\pi r^3 \times 10^9}$$

$$\text{or, or, } R^3 = 10^9 \times r^3$$

$$\text{or, } r^3 = \frac{R^3}{10^9} = \frac{(1.465 \times 10^{-3})^3}{10^9}$$

$$\text{or, } r^3 = 3.144 \times 10^{-18}$$

$$\text{or, } r = 5.32 \times 10^{-9}$$

Increase in surface area

$$A = 10^9 \times 4\pi r^2 - 4\pi R^2$$

$$= 10^9 \times 4\pi (10^9 r^2 - R^2)$$

$$= 4\pi \{ 10^9 (5.32 \times 10^{-9})^2 - (1.465 \times 10^{-3})^2 \}$$

$$= 4\pi$$

Q] Calculate the work done in spraying a spherical drop of mercury of radius  $10^{-3}$  m into a million drops of equal size. Surface tension of mercury =  $550 \times 10^{-3} \text{ Nm}^{-1}$ .

⇒ Here,

The radius of mercury,  $R = 10^{-3}$  m.

Total drops =  $10^6$

So,

$$\frac{4}{3} \pi R^3 = 10^6 \times \frac{4}{3} \pi r^3$$

$$\text{or, } R^3 = 10^6 \times r^3$$

$$\text{or, } R = 10^2 \times r$$

$$\text{or, } r = \frac{R}{100} = \frac{10^{-3}}{10^2} = 10^{-3} \times 10^{-2}$$

$$r = 10^{-5} \text{ m.}$$

Increase in surface area

$$A = 10^6 \times 4\pi r^2 - 4\pi R^2 = 4\pi (10^6 \times r^2 - R^2)$$



$$A = 4\pi \left\{ 10^6 \times (10^{-5})^2 - (10^{-3})^2 \right\}$$

$$= 4\pi \times 99 \times 10^{-6} \text{ m}^2$$

$$= 1.244 \times 10^{-3}$$

$$\text{Work done} = T \times A$$

$$= 550 \times 10^{-3} \times 1.244 \times 10^{-3}$$

$$= 6.842 \times 10^{-4}$$

Calculate the amount of energy needed to break a drop of water of diameter  $2 \times 10^{-3} \text{ m}$  into  $10^9$  droplets of equal size. Surface tension of

water  $72 \times 10^{-3} \text{ Nm}^{-1}$ .

⇒ Here,  $\frac{R}{r} = \frac{10^9}{10^9} = 1$

The diameter is  $= 2 \times 10^{-3} \text{ m}$

So, Radius,  $R = 10^{-3} \text{ m}$ .

Droplets of equal size  $= 10^9$

Surface Tension,  $T = 72 \times 10^{-3} \text{ Nm}^{-1}$

#  
So,

$$\frac{4}{3} \pi R^3 = 10^9 \times \frac{4}{3} \pi r^3$$

$$\text{Or, } R^3 = 10^9 \times r^3$$

$$\text{Or, } r^3 = \frac{R^3}{10^9}$$

$$\text{Or, } r = \frac{R}{10^3} = \frac{10^{-3}}{10^3} = 10^{-3-3} = 10^{-6} \text{ m.}$$

Also,

Increase in surface Area,

$$A = 10^9 \times 4\pi r^2 - 4\pi R^2$$

$$= 4\pi (10^9 \times r^2 - R^2)$$

$$= 4\pi \left\{ 10^9 \times (10^{-6})^2 - (10^{-3})^2 \right\}$$

$$= 4\pi \left( (10)^{9-12} - 10^{-6} \right)$$

$$= 4\pi (10^{-3} - 10^{-6})$$

$$= 1.255 \times 10^{-2}$$



Excess pressure,

$$1 \text{ side (Inside/outside)} = \frac{2T}{r}$$

$$2 \text{ sides (Both inside + outside)} = \frac{4T}{r} \quad \square$$

∴ Amount of energy =  $A \times T$

$$= (1.255 \times 10^{-2} \times 7.2 \times 10^{-3}) \text{ J}$$

$$= 9.039 \times 10^{-4} \text{ J}$$

(Ans)

Excess pr.  $\square$  An air bubble of radius 0.1 mm is situated just below the surface of water. Calculate the gauge pressure (excess of pressure) inside the air bubble. Surface

Tension of water =  $7.2 \times 10^{-2} \text{ N/m}$

⇒ Excess of pressure =  $\frac{2T}{r}$

$$= \frac{7.2 \times 10^{-2} \times 2}{(0.1 \div 10^{-3})}$$

$$= \frac{7.2 \times 10^{-2} \times 2}{(0.1 \div 10^{-3})} = 1.44 \times 10^3$$

$$= 1440 \text{ Nm}^{-2}$$

(Ans)

In an experiment for determining the surface tension of water by capillary rise, a capillary ~~rise~~ tube of diameter 1 mm is used. The height of water in the capillary tube was found to be 3 c.m. Calculate the surface tension of water. Take density of water  $10^3 \text{ kgm}^{-2}$ .

Note!-

$$\text{Excess pressure} = h\rho g$$

$$\therefore \frac{2T}{r} = h\rho g$$

The pressure of liquid column

Here,

$$\text{Diameter} = 1 \text{ mm}$$

$$\text{Radius, } r = 0.5 \text{ m.m} = \frac{0.5}{1000}$$

$$= 5 \times 10^{-4} \text{ m.}$$

$$\text{The height of water, } h = 3 \text{ c.m.}$$

$$= 3 \times 10^{-2} \text{ m.}$$

$$\text{Density of water, } \rho = 10^3 \text{ kgm}^{-2}$$



So, the difference in water level,

$$h = (h_1 - h_2)$$

$$= (29.39 - 14.694) \text{ m}$$

$$= 14.696 \text{ m}$$

$$= (2.939 - 1.47) \times 10^{-3} \text{ m}$$

$$= 1.469 \times 10^{-3} \text{ m}$$

(Ans)

Q A glass tube of internal radius  $5 \times 10^{-3} \text{ m}$  is dipped vertically into a vessel containing mercury such that the lower end of the tube is  $10^{-2} \text{ m}$  below the surface of mercury. Calculate the gauge pressure of air inside the tube to blow a hemispherical bubble at the lower end of the tube. Surface tension

of mercury =  $3.5 \times 10^{-1} \text{ Nm}^{-1}$ . Density of mercury =  $1.36 \times 10^4 \text{ kg m}^{-3}$ .

=> Here,

Internal radius of the glass tube,  $r = 5 \times 10^{-3} \text{ m}$ .

Lower end of that tube,  $h = 10^{-2} \text{ m}$ .

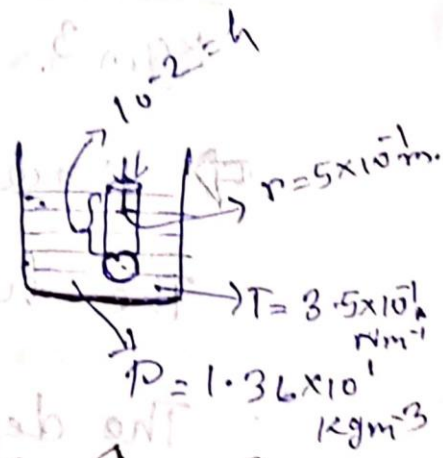
Surface tension of mercury,  $\Gamma = 3.5 \times 10^{-1} \text{ Nm}^{-1}$

Density of mercury,  $\rho = 1.36 \times 10^4 \text{ kg m}^{-3}$

Gravity,  $g = 9.8 \text{ ms}^{-2}$

Gauge pressure = ?

~~10~~  
Gauge pressure =  $\frac{2\Gamma}{r} + h\rho g$



$$= \frac{2 \times 3.5 \times 10^{-1}}{5 \times 10^{-3}} + 10^{-2} \times 1.36 \times 10^4 \times 9.8$$

$$= \frac{2 \times 3.5 \times 10^{-1} + (5 \times 10^{-3})(10^{-2} \times 1.36 \times 10^4 \times 9.8)}{5 \times 10^{-3}}$$

$$= \frac{2.7328 + 1472.8}{5 \times 10^{-3}} \text{ Nm}^{-2}$$

(Ans)



□ In a capillary tube, water rises to a height of 0.1 m. In the same capillary tube mercury is depressed by  $3.42 \times 10^{-2}$  m.

Angle of contact for water = 0

Angle of contact for mercury = 135°

Calculate the surface tension of mercury given the surface tension of water as  $72 \times 10^{-3} \text{ Nm}^{-1}$ , Density of mercury =  $13.6 \times 10^3 \text{ kgm}^{-3}$ .

⇒ Here, Density of  $\rho = 1000 \text{ kgm}^{-3}$

The height of water,  $h_1 = 0.1 \text{ m}$ .

The depression of mercury =  $3.42 \times 10^{-2} \text{ m}$ .

At Angle of Contact,

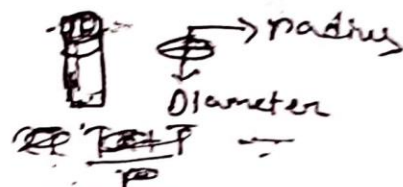
For water,  $\theta_1 = 0^\circ$

For mercury,  $\theta_2 = 135^\circ$

Surface Tension of water  $T_1 = 72 \times 10^{-3} \text{ N/m}$

Density of mercury,  $\rho_2 = 13.6 \times 10^3 \text{ kgm}^{-3}$

(Depression) ऊर्ध्व मान रहे  
(negative)



The surface tension of mercury =  $T_2$ ,  $T_1 = ?$

Note :-

सूत्रानुसार,  $\frac{2T \cos \theta}{r} = h \rho g$

किन्तु यहाँ, Angle of contact  $\theta$  का  $\theta$  माना जाये

माना जायेगा, अर्थात्,  $\frac{2T \times \cos \theta}{r} = h \rho g$

Also,

For water,  $T_1 = \frac{h_1 \rho_1 g \cdot r}{2 \cos \theta_1}$  — (i)

For mercury,  $T_2 = \frac{h_2 \rho_2 g r}{2 \cos \theta_2}$  — (ii)

Here,

$$\frac{T_2}{T_1} = \left( \frac{h_2}{h_1} \right) \times \left( \frac{\rho_2}{\rho_1} \right) \times \left( \frac{\cos \theta_1}{\cos \theta_2} \right)$$

$$\text{Or, } T_2 = T_1 \left( \frac{h_2}{h_1} \right) \times \left( \frac{\rho_2}{\rho_1} \right) \times \left( \frac{\cos \theta_1}{\cos \theta_2} \right)$$

$$\text{Or, } T_2 = 72 \times 10^{-3} \times \frac{-3.42 \times 10^{-2} \times 13.6 \times 10^3 \times \cos 0^\circ}{0.1 \times 1000 \times \cos 135^\circ}$$

$$\text{Or, } T_2 = 4.73 \times 10^{-7} \quad 4.73 \times 10^{-1} \quad (\text{Ans})$$



In Jaeger's experiment, a capillary tube of internal diameter  $5 \times 10^{-4} \text{ m}$  dips  $3 \times 10^{-2} \text{ m}$  inside water contacted in a beaker. The difference in level of water manometer when the bubble is released in  $0.09 \text{ m}$ . Calculate the surface tension of water.

$\Rightarrow$  Here, Density of  $P_1$  &  $P_2 = 10^3 \text{ kg m}^{-3}$

The diameter of a capillary tube =  $5 \times 10^{-4} \text{ m}$   
 Radius,  $r = 2.5 \times 10^{-4} \text{ m}$

Height of water,  $h = 3 \times 10^{-2} \text{ m}$

The difference in level of water manometer,  $H = 0.09 \text{ m}$ .

Surface Tension,  $T = ?$

We know,

$$\frac{2T}{r} + h P_1 g + P = P + H P_2 g$$

$$\text{Or, } \frac{2T}{r} = H\rho_2 g - h\rho_1 g$$

$$\text{Or, } T = \frac{r(H\rho_2 g - h\rho_1 g)}{2}$$

$$\text{Or, } T = \frac{(2.5 \times 10^{-4}) (0.09 \times 10^3 \times 9.8 - 3 \times 10^3 \times 9.8)}{2}$$

$$\text{Or, } T = \frac{(2.5 \times 10^{-4} \times 9.8)}{2} \times (0.09 \times 10^3 - 3 \times 10^3)$$

$$= 0.1102$$

$$\text{Or, } T = 1.102 \times 10^{-1} \text{ N/m}$$

$$\text{Or, } T = 0.1102 \text{ N/m}$$

$$\text{Or, } T = 73.5 \times 10^{-3} \text{ N/m}$$

$$\text{(Ans)}$$

This is only for liquid

A fluid in motion possesses various forms of energy. The kinetic energy, potential energy and gravitational energy. In the case of ideal liquids it is assumed that the viscous forces are completely absent.



viscous → जादू

Incompressible → अमरुतम

## Lesson - 02 (Final)

### Fluid Dynamics and viscosity

1. Eq<sup>n</sup> of c

2. B. Theorem

3. viscosity

4. Stoke's law

#### Introduction:-

In the case of a liquid at rest, the hydrostatic pressure at any point inside the liquid is given by,

$$P = h \rho g$$

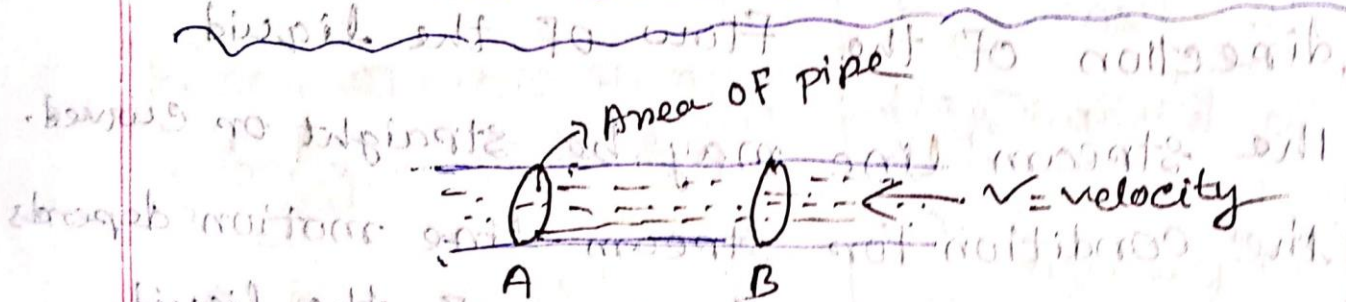
[ h = height of water ]

→ This is only for liquid

A fluid in motion possesses various forms of energy like kinetic energy, potential energy and gravitational energy. In the case of ideal liquids, it is assumed that the viscous forces are completely absent

and the liquid is highly incompressible.

Stream line motion and Rate of Flow:



$v =$  velocity of water.

$a =$  Area of ~~pressure~~ pipe / Area of cross section

যদি  $Q$  পরিমাণ Flow হবে সমস্ত

$$Q = v \cdot t$$

$$Q = v \cdot t \cdot a \text{ or } Q = v \cdot a$$

$\Rightarrow$  ~~the~~ volume of liquid per unit time,

$$\Rightarrow \frac{v \cdot t \cdot a}{t} = v \cdot a \Rightarrow (\text{velocity of flow} \times \text{area of cross section})$$

• যদি কোনাে ন্যূনতম  $Q$  বা  $v$  বা  $a$  সমান হলে liquid

Flow হয় তবে সেই Stream line or steady.

Stream line motion <sup>শূন্য</sup> upper surface এর velocity

সর্বোচ্চ বেগ  $v$  এর Lower surface এর velocity

সর্বোচ্চ  $v$  হয়।



In case of stream line motion, at any instant, the tangent to curve gives the direction of the flow of the liquid.

The stream line may be straight or curved. The condition for stream line motion depends on the velocity of flow of the liquid.

If the velocity does not exceed a certain limiting value called the critical velocity.

If the velocity is <sup>not</sup> higher than the

critical velocity, the flow is said to be stream line.

And if it is higher than critical velocity the motion is said to be turbulent.

For laminar flow, Reynold's number  $R$  should be less than 2000. According to Reynold, the critical velocity of a liquid is given

by,

$$Re = \frac{v \rho D}{\eta}$$



OR, 
$$v_c = \frac{Re_c \eta}{\rho D}$$

Reynold এর নামের মত যাড়ের  
velocity ও তত যাড়ের।

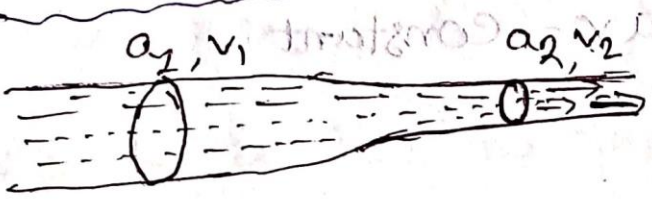
<sup>critical</sup>  
 $v_c$  = velocity of flow  
 $Re$  = Reynold's number  
 $\rho$  = Density of the fluid  
 $D$  = Diameter of the tube  
 $\eta$  = Coefficient of viscosity of the liquid

Turbulent flow the path of the particles during their motion is zig-zag.  
 তাই একই layer এ flow হয় না, এই particles সূত্র ভিন্ন ভিন্ন layer-এ flow হয়।

Example:-

The flow of river's water is a stream line motion in natured general time.

Equation of continuity:-



এই কান্ডগায় অর্থাৎ  $a_1$  এর area of pipe বড় কিন্তু  $a_2$  ছোট। আর ছোট cross section



velocity को

$$\frac{A_1 v_1}{A_2} = v_2$$

वेग,

$$\boxed{a_1 > a_2}$$
  
$$\boxed{v_2 > v_1}$$

velocity

सततता

आदि को, equation of continuity,

As there is no accumulation (संग्रह) of the liquid at any point, the amount of liquid flowing per second is the same at all cross-sections of the tube.

Amount of liquid flowing per second,  $A = a_1 v_1$

Amount of " " " " " "  $B = a_2 v_2$

So,  $\boxed{a_1 v_1 = a_2 v_2}$  is the equation of continuity.

In general,  $av = \text{constant}$



## Energy of a liquid in Motion:

1] Liquid motion is called kinetic energy.

Mathematically, kinetic energy =  $\frac{1}{2}mv^2$

But the kinetic energy per unit weight is called velocity head and is equal to

$$\Rightarrow \frac{\frac{1}{2}mv^2}{mg} \left( \because w = \text{weight} = mg \right)$$

$$\text{or, } \frac{\frac{1}{2}v^2}{g} \quad \text{or, } \frac{v^2}{2g}$$

Liquid energy per unit weight

Calculation

$$\text{So, } \boxed{\text{velocity head} = \frac{v^2}{2g}}$$

## 2] Potential Energy:-

This energy is due to the position of the liquid with respect to the ground level.



Exerted  $\rightarrow$  ऊर्जा

So, we know, P.E =  $mgh$

$$\text{So, potential head} = \frac{mgh}{mg} = h$$

The potential energy per unit weight

3) pressure Energy:-

The energy is due to the pressure exerted (ऊर्जा) on the liquid while it is flowing.

$$\text{This energy is } P \times V = P \times \text{volume}$$

↓  
pressure

So, The pressure energy per unit weight

$$= \frac{PV}{mg}$$

$$\text{But we, } P = \frac{m}{V} \therefore \frac{1}{\rho} \frac{1}{\rho} = \frac{V}{m}$$

$$\text{So, pressure head} = \frac{P}{\rho g}$$

So, now if we add all ~~three~~ ~~ener~~ values of three energies, we will get total energy per unit mass at any point is

Given by :-

$$\left( \frac{v^2}{2} + gh + \frac{P}{\rho} \right)$$

[when the flow ~~of~~ the stream line]

If we calculate with total energy per unit weight :-

$$\left( \frac{v^2}{2g} + h + \frac{P}{\rho g} \right)$$

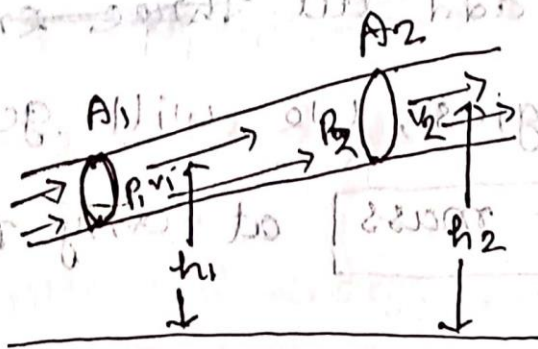
= Constant

### Bernoulli's Theorem :-

যদি liquid এর flow যদি steady বা non-turbulent হয় তবে energy কুলোব মোটামুটি

সমান হবে।





The motion of the liquid is stream line.

Let the pressure of the liquid at the

Cross-section at  $A_1 = a_1$  and at  $A_2 = a_2$

The velocity of flow of the liquid at  $A_1$

is  $v_1$  and at  $A_2$  is  $v_2$ .

work done per second on the liquid

entering  $A_1$ ,  $w_1 = P_1 a_1 v_1$

leaving,  $A_2$ ,  $w_2 = P_2 a_2 v_2$

We know  
 work,  $w = FS$   
 w. done per time,  $\frac{w}{t} = \frac{FS}{t}$   
 Also,  
 pressure,  $P = \frac{F}{A}$ ,  $F = PA$   
 so,  $\frac{w}{t} = \frac{PA \cdot S}{t} = [P \cdot A \cdot v]$

Net work done on the liquid,

$$W = w_1 - w_2 = P_1 a_1 v_1 - P_2 a_2 v_2$$

But,  $a_1 v_1 = a_2 v_2$  [Cause it is in stream line equation of continuity]

$$\therefore W = (P_1 - P_2) a_1 v_1 = (P_1 - P_2) a_2 v_2$$

This work done on the liquid contributes  
 For (the changes in) kinetic energy & gravit-  
 ational energy.

Note:-

Gravitational energy

From potential energy,  $P = mgh$

we also know,  $m = \rho V \rightarrow V = \text{volume}$ .

$[\rho = \frac{m}{V}]$

we know, volume,  $V = a \cdot l$  (area · length)

now,  $\frac{mgh}{t} = \frac{\rho V gh}{t}$  [ $\because m = \rho V$ ]

now,  $= \frac{\rho \cdot a \cdot l \cdot gh}{t} \Rightarrow \rho a g h \cdot \frac{l}{t} \Rightarrow \rho a g h \cdot v$   
velocity

Now, change in

gravitational energy,  $E_1 = (\rho_1 V_1) \rho g \cdot (h_2 - h_1)$

Kinetic energy,  $E_2 = \frac{1}{2} m v^2$  (This is what we know)

$= \frac{1}{2} \rho \cdot a_1 v_1 \cdot v^2$  total velocity

$= \frac{1}{2} (a_1 V_1) \rho (v_2^2 - v_1^2)$



$$\text{So, } W = E_1 + E_2$$

$$\text{Or, } (P_1 - P_2) a_1 v_1 = (a_1 v_1) \rho \cdot g (h_2 - h_1) + \frac{1}{2} (a_1 v_1) \rho (v_2^2 - v_1^2)$$

$$\text{Or, } P_1 + \rho g h_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2} \rho v_2^2$$

$$\text{Or, } \frac{v_1^2}{2} + g h_1 + \frac{P_1}{\rho} = \frac{v_2^2}{2} + \frac{P_2}{\rho} + g h_2$$

$$\text{Or, } \frac{v_1^2}{2} + g h + \frac{P_1}{\rho} = \text{Constant} \quad \text{--- (ii)}$$

This equation represents Bernoulli's equation

Dividing equation (ii) by  $g$ ,

$$\left( \frac{v^2}{2g} \right) + h + \left( \frac{P_1}{\rho g} \right) = \text{Constant}$$

Here,

$\frac{v^2}{2g}$  = The velocity head ✓

$h$  = gravitational head ✓

$\frac{P}{\rho g}$  = pressure head ✓

Special case, all venturimeters:

When it is a horizontal pipe,

$h$  is constant,

$$\frac{v^2}{2} + \frac{P}{\rho} = \text{constant}$$

$$\text{or, } P + \frac{1}{2} \rho v^2 = \text{constant}$$

$P$  = Static pressure

$\frac{1}{2} \rho v^2$  = Dynamic pressure

Viscosity (Effect)

A venturimeter is a horizontal tube. It has different cross sections at A and B. It is used to find the rate of flow of a liquid when the motion of the liquid crossing through any cross-section of the pipe is constant.

It means that, the amount of liquid crossing per second at A is equal to the amount of liquid crossing per second at B. But



$$\frac{P_1}{\rho} - \frac{P_2}{\rho} - \frac{P_2}{\rho} - \frac{P_1}{\rho} = gh - gh = g^x$$

$$= -gh$$

the area of cross section at A is large.

### 7. Law of hydrostatic pressure:-

The difference in pressure between two points A and B can be calculated by applying Bernoulli's principle.

The liquid in the vessel is at rest.

Therefore,  $v=0$ . The total energy per unit mass at A,

$$= g(h+x) + \frac{P_1}{\rho} \quad \text{--- (i)}$$

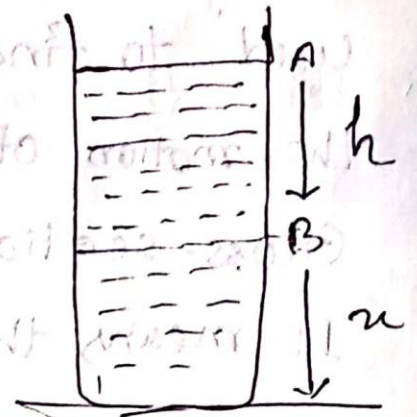
Total energy per unit mass at B,

$$= gx + \frac{P_2}{\rho} \quad \text{--- (ii)}$$

Equating (i) & (ii)

$$g(h+x) + \frac{P_1}{\rho} = gx + \frac{P_2}{\rho}$$

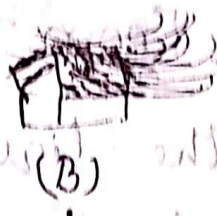
$$\boxed{P_2 - P_1 = \rho g h}$$



## 8] Blowing of Roofs:

Due to wind, storm or cyclone, the roofs are blown off. When a high velocity wind blows over the roof, there is considerable lowering of pressure on the roof.

As the pressure on the lower side of the roof is higher, roofs are easily blown off without damaging the walls of the building.



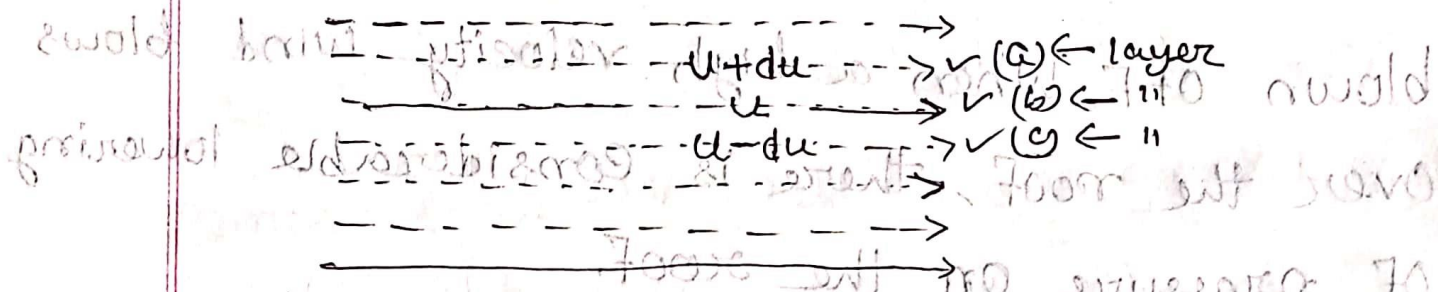
Due to wind, storm or cyclone, the roofs are blown off.

lower side of the roof has high pressure. So, roofs are easily blown off without damaging the building.



Streamline  $\rightarrow$  Fluid Flow but free from turbulent.

### viscosity (मायुज)



$\Rightarrow$  Whenever a liquid flows on a horizontal surface, the velocities of the different layers of the liquid parallel to the fixed surface are different and increase with the distance from the fixed surface.

If the motion of the liquid is streamline, the layer of the liquid in contact with the fixed surface is stationary. The velocity of any layer increases with the distance from the fixed layer. If any two layers are considered, the upper layer tends to accelerate the motion of the lower layer and the lower layer tends to retard the motion of the upper layer. The



↳ two layers together tends to destroy their relative motion. As if there is a backward tangential force. (যদি তরলের হাতি আন্তরিক হয়

অদ্যাবধি বর্তমান স্তরের সাথে শাষণ তরল হয় স্থির।  
মতই অদ্যাবধি বর্তমান স্তর হয়ে দৃষ্টি বাড়ে ততই স্তরের  
তরলের বেগ বৃদ্ধি পায়। যদি দুইটি স্তরের স্তরকে  
ধরা হয়, উপরের স্তরটি নিচের স্তরের বেগকে বৃদ্ধি  
করতে চাইবে এবং নিচের স্তরটি উপরের স্তরের বেগকে  
কমতে চাইবে। এর কারণ, একে অপরকে হাতিকে নিজে  
নিজের হাতিকে অমান করতে চাইবে। ~~সেই~~ ~~আদর~~ ~~এর~~  
আন্তরিক অথবা একে অপরকে আন্তরিক হাতিকে  
ধ্বংস করতে চাইবে)।

An external force is required to overcome this backward drag and to maintain the relative velocity between the different layers of the liquid. This property by virtue (বৃত্ত) of which



a liquid opposes the relative motion between the different layers is called viscosity or internal friction.

Generally, thin liquids like alcohol, water, spirit etc. are less viscous.

Whereas, thick liquids like coal tar, castor oil, glycerine etc. are more viscous.

• According to Newton, the backward tangential force  $F$  on any layer is dependent on,

(i)  $F \propto A$ , the area of the layer.

(ii)  $F \propto \frac{du}{dn}$ , the velocity gradient at the layer,

$$F \propto A \cdot \frac{du}{dn} \rightarrow \left[ \begin{array}{l} \text{Distance এর সাথে তার} \\ \text{velocity এর পরিবর্তন} \end{array} \right]$$

$$\text{or, } F = -\eta A \frac{du}{dn}$$

□ Here,

□  $\eta$  is the coefficient of viscosity of the fluid and  $\frac{du}{dn}$

(ii)  $\frac{du}{dx}$  is called the viscosity gradient or change of velocity with distance.

(iii) The negative sign shows that the force is acting opposite to the direction of velocity.

\*  $\eta$  is called the coefficient (ضمان) of viscosity of the fluid and  $\frac{du}{dx}$  is called

the velocity gradient. The negative sign shows that the force is acting opposite to the direction of velocity.

If,  $A=1$  &  $\frac{du}{dx}=1$  then,  $F=-\eta$

Therefore, the coefficient of viscosity is defined as the tangential force required to maintain a unit velocity gradient between two layers of area one unit each.



Units, If  $A = 1 \text{ sq. cm}$ ,

$$\frac{du}{dy} = \frac{1 \text{ cm/s}}{1 \text{ cm}}$$

and  $F = 1 \text{ dyne}$ , then [CGS unit]

$$\eta = \frac{F}{A \cdot \frac{du}{dy}} \rightarrow \frac{\text{dyne}}{\text{cm}^2 \cdot \text{s}^{-1}}$$

Numerically,  $\eta = 1 \text{ poise}$

The coefficient of viscosity (संयोजक गुणक) of a liquid is one poise, if a force of 1 dyne is required to maintain a velocity gradient of one unit between two layers of area 1 sq. cm each.

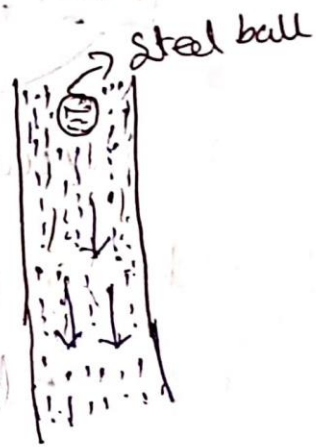
Dimensions of coefficient of viscosity:-

$$[\eta] = \frac{[F]}{[A] \left[ \frac{du}{dy} \right]} = \frac{[MLT^{-2}]}{[L^2][T^{-1}]} = [ML^{-1}T^{-1}]$$

Unit of  $\eta$  in SI is  $\text{kg/m-s}$  or  $\text{N-s/m}^2$

### Stoke's law:-

When a small ball is left gently on the surface of a long vertical column of a viscous liquid, the ball moves vertically downward. Initially, the ball is accelerated due to gravity. The motion of the ball is opposed by the viscosity of the liquid. Consequently, the resultant force acting on the ball becomes zero and it moves with a constant velocity called the terminal velocity  $v$ .



The resistive force  $F$ , acting on the ball depends on,

- (i) The radius of the ball,  $r$
- (ii) The terminal velocity,  $v$
- (iii) Coefficient of viscosity,  $\eta$ .



Expressing dimensionally,  $F$  &  $r, v, \eta$

$$[F] = k [r^a] [v^b] [\eta^c] \quad \text{--- (1)}$$

$$[MLT^{-2}] = [L]^a [LT^{-1}]^b [ML^{-1}T^{-1}]^c$$

$$[MLT^{-2}] = [M]^c [L]^{a+b-c} [T]^{-b-c}$$

$\downarrow$                        $\downarrow$                        $\downarrow$   
 $[M]^1$                        $[L]^1$                        $[T]^{-2}$

Equating the powers.

$$c = 1$$

$$-b - c = -2$$

$$-b = -1$$
$$b = 1$$

$$a + b - c = 1$$

$$a = 1$$

Substituting these values in equation (i)

$$F = k r v \eta$$

But,  $k = 6\pi$   $\rightarrow$   $[k] = [ML^{-1}T^{-1}]$   $\rightarrow k = 6\pi$

$$\text{Force, } F = 6\pi \eta r v$$

Coefficient of viscosity of a liquid  
can be determined by Stoke's formula,

Suppose,

Radius of the ball =  $r$ .

Terminal velocity =  $v$

Coefficient of viscosity =  $\eta$

Density of the ball =  $\rho$

Density of the liquid =  $\sigma$

Net weight of the ball =  $\frac{4}{3} \pi r^3 (\rho - \sigma) g$  — (i)

Upward force due to viscosity, =  $6 \pi \eta r v$

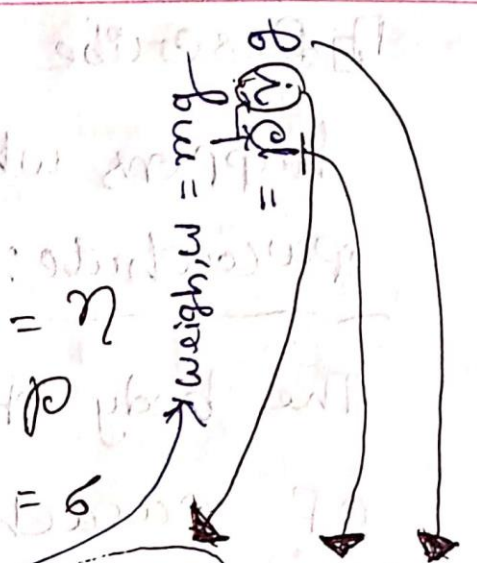
$$6 \pi \eta r v = \frac{4}{3} \pi r^3 (\rho - \sigma) g \quad \text{--- (ii)}$$

Terminal velocity,  $v = \frac{2r^2 (\rho - \sigma) g}{9 \eta}$

co. viscosity,  $\eta = \frac{2r^2 (\rho - \sigma) g}{9v}$

Hence  $\eta$  can be determined.

Stoke's law is applicable to determine coefficient of viscosity of highly viscous liquids.





Describe with liquid's law, what happens when someone jumps with parachute:-

The body of jumper & the direction of parachute is the reason why someone lands success

বাস্তব মা প্যারাসুটে আঁকে থাকে তার direction উদ্বার দিকে।

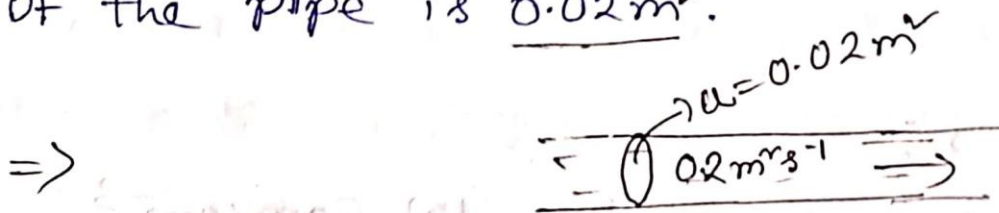
এখন উৎসের velocity reaction মতন same হবে তখন তুমি একটা fixed velocity পৌঁছো থাকবে আর তুমি আমরা বলি

Terminal velocity, এখন আমরা এমন মনে হবে যে সে নির্দিষ্ট বেগে স্থিরভাবে land করবে।

## Math Solve

Example:- 7.1 :- (Theory : Stream line motion)  
and Rate of Flow  
(वास्तविक)

Water flows through a horizontal pipe  
line of varying cross section at the rate  
of  $0.2 \text{ m}^3 \text{ s}^{-1}$ . Calculate the velocity of water  
at a point where the area of cross section  
of the pipe is  $0.02 \text{ m}^2$ .



Here,

Area of cross section =  $0.02 \text{ m}^2$

Rate of Flow =  $0.2 \text{ m}^3/\text{s}$

We know, velocity =  $v$ ?

Ratio of Flow =  $av$

$$\text{or, } v = \frac{\text{Ratio of Flow}}{a}$$

$$= \frac{0.2}{0.02} = 10 \text{ m s}^{-1}$$

(Ans)



7.2] A pitot tube is fixed on the wing of an aeroplane to measure the speed of the aeroplane. The tube contains a liquid of density  $800 \text{ kg m}^{-3}$ . The difference in level between the two limbs is  $0.5 \text{ m}$ . Density of air =  $1.293 \text{ kg m}^{-3}$ . Calculate the speed of the aeroplane.



Here,

The density of liquid,  $\rho_1 = 800 \text{ kg m}^{-3}$

The difference in level between two limbs,  $H = 0.5 \text{ m}$ .

Density of air,  $\rho_2 = 1.293 \text{ kg/m}^3$

velocity of plane,  $v = ?$

We know,

$$\frac{1}{2} \rho_2 \times v^2 = \rho_1 \times H \times g$$

$$\text{or, } v^2 = \frac{2 \rho_1 \times H \times g}{\rho_2}$$

$$\text{or, } v = \sqrt{\frac{2P_2 \times 11 \times g}{\rho_2}} = \sqrt{\frac{2 \times 1000 \times 0.5 \times 9.8}{1000}} = 77.87 \text{ ms}^{-1}$$

(Ans)

Example - 7.4:-

Calculate the speed at which the velocity head of a stream of water is equal to 0.5m of Hg.

→ (note:- The standard atmospheric pressure of 760 mm in height, exerted when the mercury density is 13.6 cm of Hg. So, the pressure of any gas is the pressure exerted by it on the walls of the container that can rise the mercury to 13.6 cm in the barometer.)

∴ we know,

$$\text{Velocity head} = \frac{vr}{2g}$$

$$0.5 \text{ m of Hg} = 0.5 \text{ m} \times 13.6 \text{ cm of water}$$



$$\text{Or, } v^2 = v.H \times 2g \times 13.6$$

$$\text{Or, } v = \sqrt{v.H \times 2g \times 13.6}$$

$$\text{Or, } v = \sqrt{0.5 \times 2 \times 9.8 \times 13.6}$$

$$\text{Or, } v = 11.54 \text{ m/s}$$

(Ans)

☐ A railway engine is fitted with a tube whose one end is inside a reservoir of water in between the rails. The other end of the tube is 4m above the surface of water in the reservoir. Calculate the speed with which the water rushes out of the proper upper end, if the engine is moving with a speed of  $108 \text{ km hr}^{-1}$ .

⇒ As the train is moving,

So there are both kinetic & potential

energy available.

So,

$$mgh_1 + \frac{1}{2}mv_1^2 = mgh_2 + \frac{1}{2}mv_2^2$$

or, here,

$$\text{or, } gh_1 + \frac{1}{2}v_1^2 = gh_2 + \frac{1}{2}v_2^2$$

$$\text{or, } g(h_1 - h_2) = \frac{1}{2}(v_2^2 - v_1^2)$$

$$\text{or, } \frac{1}{2}v_1^2 = g(h_2 - h_1) + \frac{1}{2}v_2^2$$

$$\text{or, } v_1^2 = 2g(h_2 - h_1) + v_2^2$$

$$\text{or, } v_1 = \sqrt{2g(h_2 - h_1) + v_2^2} \quad \text{--- (i)}$$

Here,

$$v_1 = ?$$

$$(\cancel{h_2 - h_1}) = 4 \text{ m. } (h_1 - h_2) = 4 \text{ or, } (h_2 - h_1) = -4$$

$$g = 9.8 \text{ m/s}^2$$

$$v_2 = 108 \text{ km/hr} = \frac{108 \times 1000}{3600} = 30 \text{ m/s}^{-1}$$

$$\text{So, } v_1 = \sqrt{2 \times 9.8 \times (-4) + (108)^2 (30)^2} = 28.66 \text{ m/s}^{-1}$$

(Ans)



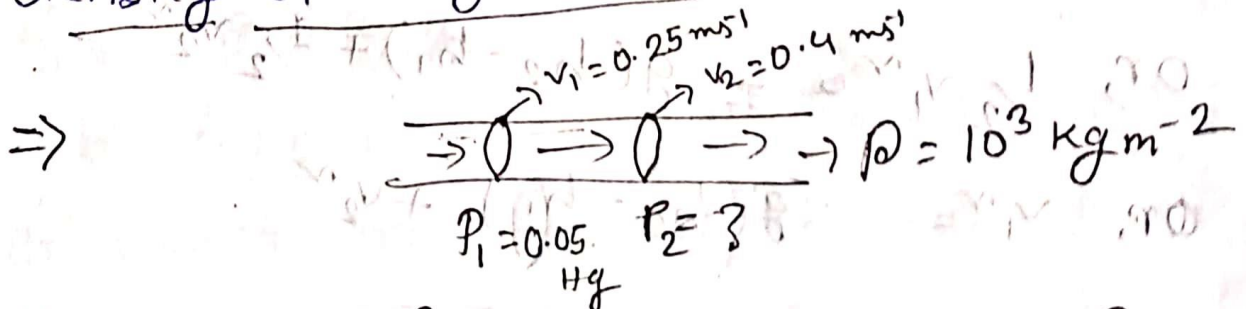
Example - 7.71 -

Water flows through a horizontal pipe line of varying cross-section. At a point where the pressure of water is 0.05 m of Hg, the velocity of flow is  $0.25 \text{ m s}^{-1}$ .

Calculate the pressure at another point

where the velocity of flow is  $0.4 \text{ m s}^{-1}$ .

Density of water is  $10^3 \text{ kg m}^{-3}$ .



Here,  $v_1 = 0.25 \text{ m s}^{-1}$ ;  $v_2 = 0.4 \text{ m s}^{-1}$   $\rho = 10^3 \text{ kg m}^{-3}$   
 pressure of water,  $P_1 = 0.05 \text{ Hg}$

$$= 0.05 \times 13.6 \times 10^3 \times 9.8 \text{ Nm}^{-2}$$

$$= 6664 \text{ Nm}^{-2}$$

we know

$$\frac{P_1}{\rho} + \frac{1}{2} v_1^2 = \frac{P_2}{\rho} + \frac{1}{2} v_2^2$$

Question:  $0.05 \text{ m}$  of Hg  
 $\text{Nm}^{-2}$

~~Or,  $P_2 = P_1 + \frac{1}{2} (v_1^2 - v_2^2) \cdot \rho$~~

~~Or,  $P_2 = 6664 + \frac{1}{2} \{ (0.25)^2 - (0.4)^2 \} \cdot 10^3$~~

~~Or,  $P_2 =$~~

~~Or,  $\frac{P_1}{\rho} = \frac{P_2}{\rho} + \frac{1}{2} v_2^2 - \frac{1}{2} v_1^2$~~

~~Or,  $\frac{P_1}{\rho} = \frac{P_2}{\rho} + \frac{1}{2} (v_2^2 - v_1^2)$~~

Or,  $\frac{P_2}{\rho} = \frac{P_1}{\rho} + \frac{1}{2} v_1^2 - \frac{1}{2} v_2^2$

Or,  $\frac{P_2}{\rho} = \frac{P_1}{\rho} + \frac{1}{2} (v_1^2 - v_2^2)$

Or,  $P_2 = P_1 + \frac{1}{2} \cdot \rho (v_1^2 - v_2^2)$

Or,  $P_2 = 6664 + \frac{1}{2} \times 10^3 \{ (0.25)^2 - (0.4)^2 \}$

Or,  $P_2 = 6664 - 48.75$

Or,  $P_2 = 6615.25 \text{ Nm}^{-2}$  (Ans)

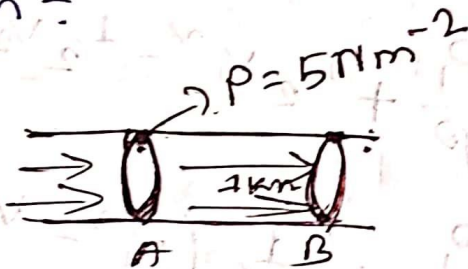
~~Or,  $P_2 = \frac{6615.25}{1.36 \times 10^3}$~~



Example:- 7.8 :-

In a horizontal pipe line of uniform area of cross section, the pressure falls by  $5 \text{ Nm}^{-2}$  between two points separated by a distance of 1 km. What is the change in kinetic energy per kg of the oil flowing at these points? Density of oil  $\rho = 800 \text{ kgm}^{-3}$ .

$\Rightarrow$



According to Bernoulli's principle,

$$\frac{P}{\rho} + gh + \frac{1}{2} v^2 = \text{Constant}$$

As the pipe line is horizontal,

$$\frac{P}{\rho} + \frac{1}{2} v^2 = \text{Constant}$$

As there are two points,

$$\frac{P_1}{\rho} + \frac{1}{2} v_1^2 = \frac{P_2}{\rho} + \frac{1}{2} v_2^2$$

$$\text{Or, } \frac{(P_1 - P_2)}{\rho} = \frac{1}{2} (v_2^2 - v_1^2) \quad \text{--- (i)}$$

Here,

$$P_1 - P_2 = 5 \text{ Nm}^{-2}$$

$$\rho = 800 \text{ kgm}^{-3}$$

$\frac{1}{2} (v_2^2 - v_1^2) =$  Changes in kinetic energy,

∴ Change in kinetic energy per k.g,

$$\frac{1}{2} (v_2^2 - v_1^2) = \frac{(P_1 - P_2)}{\rho}$$

$$= \frac{5}{800} \text{ J/kg}$$

$$= 6.25 \times 10^{-3} \text{ J/kg}$$

(Ans)

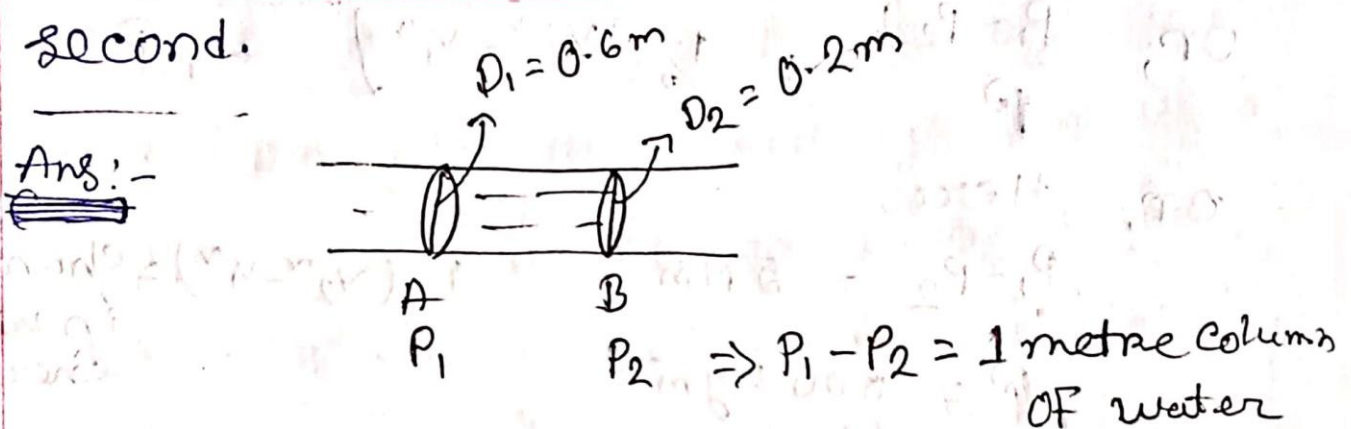
**Example:- 7.10** (प्रश्नमा आछ)

Water is flowing through a horizontal pipe line. At two points A and B. The diameters are 0.6m and 0.2m. The pressure difference between the points A and B is 1 metre column of water. Calculate the volume of water flowing per



second.

Ans: -



Here,

Diameter,  $D_1 = 0.6 \text{ m}$ .

$D_2 = 0.2 \text{ m}$ .

The pressure difference,  $(P_1 - P_2) =$

$= 1 \text{ metre column of water.}$

$$= 1 \times 10^3 \times 9.8$$

$$= 9.8 \times 10^3 \text{ Nm}^{-2}$$

The volume of water,  $V = ?$

$\Rightarrow$  For A,

$$\text{Rate of flow} = a_1 a_2 \sqrt{\frac{2(P_1 - P_2)}{\rho(a_1^4 - a_2^4)}}$$

$$\text{Here, } a_1 = \frac{\pi \times (0.6)^2}{4} = 0.09 \pi \text{ m}^2$$

$$a_2 = \frac{\pi (0.2)^2}{4} = 0.01 \pi \text{ m}^2$$

$$P_1 - P_2 = (1 \times 10^3 \times 9.8) \text{ Nm}^{-2}$$

$$a_1^2 - a_2^2 = 8 \times 10^{-9} \pi^2$$

The horizontal range,

$$l = v \times t = v \times \sqrt{\frac{2h_2}{g}} = (\sqrt{2gh_1}) \sqrt{\frac{2h_2}{g}}$$

$$= 2\sqrt{h_1 h_2}$$

$$h_2 = H - h_1$$

$$l = 2\sqrt{h_1(H-h_1)}$$

$$l^2 = 4h_1H - 4h_1^2$$

∴ Differentiating,

$$2l \left( \frac{dl}{dh_1} \right) = 4H - 8h_1$$

For  $l$  to be maximum,  $\frac{dl}{dh_1} = 0$

$$\therefore 4H - 8h_1 = 0$$

$$h_1 = \frac{H}{2}$$

$$h_2 = \frac{H}{2}$$

and, therefore, for the range of the liquid to be maximum, the orifice must be at half the height of the liquid column.



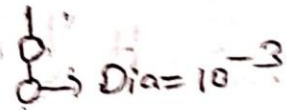
Example - 7.11:-

(a) Find the limiting velocity of a rain drop. Assume, diameter =  $10^{-3}$  m.

Density of air relative to water =  $1.3 \times 10^{-3} \text{ kg m}^{-3}$

Coefficient of viscosity of air =  $1.81 \times 10^{-8}$  S.I. units

Density of water, =  $10^3 \text{ kg m}^{-3}$



⇒ According to Stoke's law,  $\rho = 1.3 \times 10^{-3}$  air density,

$$6\pi\eta rv = \frac{4}{3}\pi r^3(\rho - \sigma)g$$

$$\text{Or, } v = \frac{2r^2(\rho - \sigma)g}{9\eta}$$
$$= \frac{2r^2\rho\left[1 - \left(\frac{\sigma}{\rho}\right)\right]g}{9\eta}$$

Here,  $r = 5 \times 10^{-4}$  m,

$$\rho = 10^3 \text{ kg m}^{-3}$$

$$\frac{\sigma}{\rho} = 1.3 \times 10^{-3}$$

$$\eta = 1.81 \times 10^{-8} \text{ S.I. units.}$$

$$v = \frac{2 \times (5 \times 10^{-4})^2 \times 10^3 [1 - 1.3 \times 10^{-3}] \times 9.8}{9 \times 1.81 \times 10^{-8}}$$

$$= 30.04 \text{ m s}^{-1} \text{ (Ans)}$$

## Lecture-03 (Final)

### Thermodynamics

(उष्मजितिया)

#### Thermodynamic System:-

A thermodynamic system is a body of matter and/or radiation confined (सीमाबद्ध) in space by walls with defined permeabilities, which separate it from its surroundings.

The surroundings may include other thermodynamic systems, or physical systems that are not thermodynamic systems. (अर्थात्, जड़ित्वलक्षण मयूँके विसे आलाचना कइए हइ वा आलाचना कइए मात्र (सिस्टम) सिस्टम)

Systems are classified as follows:-

#### Isolated system (विच्छिन्न व)स्था):

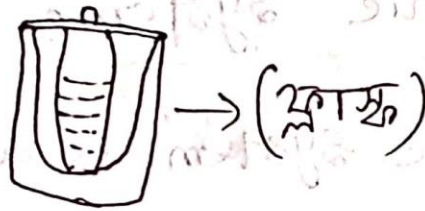
If there is no interchange of energy or matter between the system and surroundings



then the system is called isolated system.

(এর ও ক্ষতি কোনটাই পরিবর্তন হতে না পারলে তাকে

Isolated System বা বিচ্ছিন্ন ব্যবস্থা বলে)



Closed System (বদ্ধ সিস্টেম):

If ~~to~~ no energy or matter ~~to~~ crosses the boundary, then the system is called closed system.

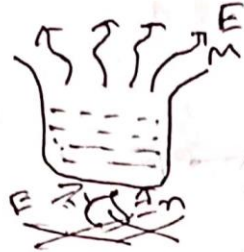
(ক্ষতি ও উষ্ণ দেওয়া যায় কিন্তু তা আর বের হতে পারে না তাকে বদ্ধ সিস্টেম বলে)



Open System: (উন্মুক্ত সিস্টেম)

If there is an interchanges of energy or matter between the system and surroundings, then the system is called

open system. (তড়ি ও কাজ উভয়ই দেওয়া ও মাঝে  
আবায় নেওয়াও মাঝে)



### Thermodynamic process

A thermodynamic process is the energetic development of a thermodynamic system proceeding (এগিয়ে মাঝে) from an initial state to a final state.

Different thermodynamic processes are:-

#### 1. Isochoric process (সমাবৃত্ত প্রক্রিয়া):

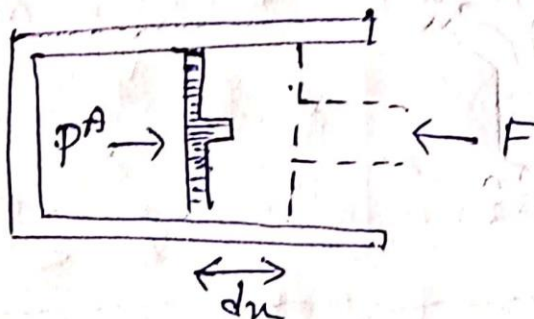
If during a process, the volume is constant then the process is called isochoric process.

#### 2. Isoobaric process (সমচাপ প্রক্রিয়া):-

যে আদ্যাতীয় প্রক্রিয়ায় সিস্টেমের চাপের কোনো



দ্বিতীয় সূত্র না তাকে সমতাপ প্রক্রিয়া বলে।



### 3. Isothermal process (সমতাপ প্রক্রিয়া):-

IF during a process the temperature is constant then the process is called isothermal process.

An isothermal process is a change of a system, in which the temperature remains constant ( $\Delta T = 0$ ). This typically occurs when a system is in contact with an outside thermal reservoir, and the change occurs slowly enough to allow the system to continually adjust to the temperature of the reservoir through heat exchange.

system whereas if the temperature of the system is slightly greater than that of the surroundings there will be a flow of heat in the opposite direction. Such process is therefore reversible process where no dissipative force is present.

### Irreversible process:-

If there is a finite temperature difference between system and surroundings the direction of the heat flow can't be reversed by an infinitesimal change

in temperature of the system and the process is irreversible.

### ☐ Concept of 1st law of Thermodynamics:-

Scientist named Joule discovered this.

### 1st law of thermodynamics:-



When heat is supplied to any system then a part of that helps to increase the internal energy of the system and the remaining part of the energy is used by the system to do external work on the environment.

(যখন কোনো সিস্টেম তাপমাত্রা বৃদ্ধি করে তখন সেই তাপমাত্রা বৃদ্ধি অংশ সিস্টেমের অভ্যন্তরীণ শক্তি বৃদ্ধিতে এবং বাকি অংশ দ্বারা সিস্টেম তাপ পরিবেশের দিকে শক্তি বাহ্যিক কাজ সম্পাদন করে)

Explanation:-

$$\textcircled{1} \Delta U = U_2 - U_1$$



The change in internal energy.

here,  $U_1$  = Internal energy of the system.

If  $Q$  amount of heat is added to the system then  $U_2$  will become the internal energy.

Thus the first law of the thermodynamics gives,

$\textcircled{2} Q = \Delta U + W \Rightarrow$  It is the mathematical form of the first law of thermodynamics.

For infinitesimal reversible process,  
the First law takes the form,

$$dQ = du + dw$$

$$\text{or, } dQ = du + p dv$$

$\downarrow$                        $\downarrow$   
 তাপ                      আয়তন

here,  
 $dQ =$  সিস্টেম কর্তৃক জোড়িত  
 তাপ  
 $du =$  অভ্যন্তরীণ ऊर्जा বৃদ্ধি  
 $dw =$  System কর্তৃক কৃত  
 কাজ

### Application of First law of thermodynamics

Molar Specific Heat :- (মোলার তাপধারণ ক্ষমতা)

The amount of heat needed to increase the temperature one Kelvin of one mole gas is called molar specific heat.

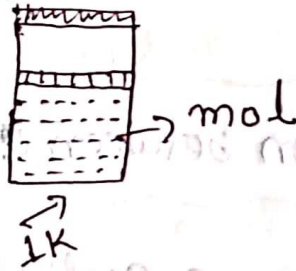
(যেহেতু পদার্থের এক মোলের এক কেলভিন বৃদ্ধি করতে প্রয়োজনীয় তাপকে ঐ পদার্থের মোলার আপেক্ষিক তাপ বা মোলার তাপধারণ ক্ষমতা বলে)।



Molar specific heat at constant pressure :-

(মিশ্র গ্যাসে ইগ্যাসের মোলার আপেক্ষিক তাপ) [ $C_p$ ]

At constant pressure, the amount of heat needed to increase the temperature one kelvin of one mole gas is called molar specific heat. অর্থাৎ [ $\pm$  mol গ্যাসের temp.  $\pm$  K (কেলভিন) বাড়াতে হলে মিশ্র গ্যাসে মতটুকু তাপ দিতে হয় তাই হলো  $C_p$ ]



Formula 2-

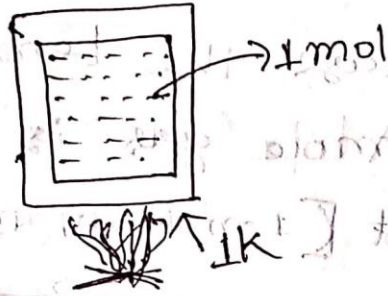
$$C_p = \frac{\Delta Q}{m \Delta T}$$

Molar specific heat at constant volume :- ( $C_v$ )

(মিশ্র গ্যাসে মোলার আপেক্ষিক তাপ)

The amount of heat needed to increase the temperature one kelvin of one mole gas is called molar specific heat.

1 mol গ্যাসের Temperature স্থির রাখতে  $1k$  (বাড়াতে) হোলো যে পরিমান তাপ দেওয়া লাগে তাহে  $C_v$  বলে)



$$C_v = \frac{dQ}{m dT} \quad \text{or,} \quad C_v = \frac{dQ}{m dT}$$

### Relation between $C_p$ and $C_v$

Let us consider a cylinder containing one mole gas. A frictionless piston is attached to the cylinder.

$P$  = pressure ;  $v$  = volume ;  $T$  = Temperature

$U$  = Internal Energy of gas,

Formulas:-

$$dQ = du + PdV \quad \text{--- (1)}$$

$$du = C_v dT \quad \text{--- (2)}$$



We know that, the amount of heat needed to increase the temperature one Kelvin of one mole gas is called molar specific heat at constant volume,  $C_v$  and increase in temperature  $dT$  pressure,

$$\text{So, } C_p = \frac{dQ}{dT}$$

$$\text{Or, } dQ = C_p dT \quad \text{--- (3)}$$

IF  $R$  is the molar gas constant, then for one mole gas we get,

$$pV = RT$$

$$\text{Or, } PdV = RdT \quad \text{--- (4)}$$

From (2), (3) and (4) we put those values into equation (1),

$$C_p dT = C_v dT + RdT$$

$$\text{Or, } C_p = C_v + R$$

$$\text{Or, } \boxed{C_p - C_v = R}$$

$$\textcircled{1} \quad dQ = dU + PdV$$

$$\textcircled{2} \quad C_v dT + PdV = 0$$

$$\textcircled{3} \quad dT = \frac{PdV + VdP}{R}$$

$$\textcircled{4} \quad C_v \left( \frac{PdV + VdP}{R} \right) + PdV = 0$$

$$\textcircled{5} \quad VdP + \gamma PdV = 0 \quad \left[ \text{where, } \gamma = \frac{C_p}{C_v} \right]$$

$$\textcircled{6} \quad \frac{dP}{P} + \gamma \frac{dV}{V} = 0$$

$$\textcircled{7} \quad \ln P + \gamma \ln V = \text{Constant}$$

$$\textcircled{8} \quad \ln P + \ln V^\gamma = \text{"}$$

$$\textcircled{9} \quad \ln(PV^\gamma) = \text{"}$$

$$\textcircled{10} \quad PV^\gamma = \text{"}$$

Here,

$$C_p > C_v \quad [\text{Always}]$$

So,

$$C_p - C_v = R \quad \text{--- (i)}$$

$$\frac{C_p}{C_v} = \gamma \quad \text{--- (ii)}$$



R is a constant = 8.314

1. नग्नमानुष = 1.67

2. " = 1.41

3. " = 1.33

} value of  $\gamma$ .

## Second Law of Thermodynamics

To convert ~~temp~~ thermal power into others we need an engine here. This is what we call as Thermo-engine.

Different scientist have stated the law in different forms:-

1) Carnot's Statement :-

No engine can be built which can extract a fixed amount of heat and will convert totally into work.

2) Clasius's Statement :-

It is impossible for a self acting machine

Unaided by external agency, convey heat from one body at a lower temperature to another body at a higher temperature

### 3. Planck's Statement :-

It is impossible to construct an engine which can extract heat continuously from a source of heat and completely transform into work.

### 4. Kelvin's Statement :-

Continuous flow of energy can't be obtained from an object cooling it than the coolest part of its surroundings.

## Heat Engine

## and Efficiency of Heat Engine

(তাপীয় ইঞ্জিন এবং তাপীয় ইঞ্জিনের দক্ষতা)

### Heat Engine :-

Any mechanism for the conversion of



heat into mechanical power is called a heat engine or any suitable device which can convert heat into mechanical work is called heat engine.

Efficiency of heat engine:-

$$\eta = \frac{W}{Q}$$

The efficiency of heat engine is defined as the ratio of work done during a cycle to the heat absorbed during the cycle.

Thus if  $W$  is the amount of work obtained from heat engine in one cycle at the expense of amount of heat, then its efficiency  $\eta$  is defined as,

$$\begin{aligned}\eta &= \frac{W}{Q} = (Q_1 - Q_2) / Q_1 \\ &= 1 - (Q_2 / Q_1)\end{aligned}$$

## Carnot's Heat Engine and its Efficiency:-

French scientist Sadi Carnot discovered this in 1824.

### In Carnot's heat engine:-

A system carried through a Carnot cycle is the prototype of all cyclic heat engines. Features that <sup>common to,</sup> they receive an input of heat at one or more higher temperatures, do mechanical work on their surroundings, and reject at some lower temperature.

When any working substance is carried through a cyclic process, there is no change in its internal energy in any complete cycle and from the First Law the net flow of heat  $Q$  into the substance in any complete cycle is equal to the work done  $W$  by the engine per cycle,



$$W = Q_1 - Q_2 = Q$$

Here,

$Q_1$  = The heat flowing in the system per cycle

$Q_2$  = The heat flowing out of the system per cycle.

Thermal efficiency :-

$$\eta = \frac{\text{work output}}{\text{heat input}} = \frac{\text{heat converted into work per cycle}}{\text{heat drawn from the heat source into the system per cycle}}$$

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} \quad \text{--- (1)}$$

[ For Carnot's engine the heat flowing in or out of the system is proportional to the temperature to the heat reservoir ]

$$Q \propto T$$

$$\text{or, } \frac{Q}{T} = \text{constant}$$

(product)  $\frac{Q}{T}$  is constant

If  $T_1$  and  $T_2$  are the temperatures of the source and sink respectively, then

$$\left(\frac{Q_1}{T_1}\right) = \frac{Q_2}{T_2}$$

$$\text{So, } \boxed{\frac{Q_1}{Q_2} = \frac{T_2}{T_1}}$$

From equation (1) we get,

$$\eta = 1 - \frac{T_2}{T_1}$$

$$\text{or, } \eta = \frac{T_1 - T_2}{T_1}$$

As efficiency (कार्यक्षमता) is expressed in term of percentage,

$$\eta = \frac{(T_2 - T_1)}{T_1} \times 100\%$$

In the same way,

$$\eta = \left(1 - \frac{Q_2}{Q_1}\right) \times 100\%$$

Note:- As,  $T_2 > T_1$ , so we can't get 100% from any engine.



If the difference between the source of heat and the receiver of heat is much then the efficiency will be much too. But ~~wouldn't~~ won't be much more than 20% or 25%, sometimes 50%.

### Entropy

The disability of converting energy of a system or unavailability of energy to convert into work is entropy.

(যেহেতু নির্দিষ্ট কাজের জন্য প্রাপ্য অক্ষমতা বা অসম্ভাব্যতা বা প্রাপ্যের জন্য কাজের অসম্ভাব্যতা বোঝায়)

⊛ It has not been possible to measure the absolute magnitude of entropy. If anybody absorbs or rejects ~~heat~~ heat then its entropy is changed.

⊛ The change of entropy is measured by the rate of absorption or rejection of heat by the system with respect to temperature.

(কোনো সিস্টেমের তাপমাত্রার পরিবর্তন ঘটে না বা  
 বর্ধিত তাপ পরিবর্তনের হার দ্বারা অনুপস্থিত  
 পরিবর্তন পরিসীমা করা হয়।)

If any system absorbs or rejects  $dQ$  amount  
 of heat at temperature  $T$ , then the  
 change of entropy

$$ds = \frac{dQ}{T} \quad \left| \begin{array}{l} dQ = \text{বর্ধিত বা বর্ধিত তাপ} \\ T = \text{বহুত্ব তাপমাত্রা} \end{array} \right.$$

$$\therefore \Delta S = \frac{\Delta Q}{T} = \frac{0}{T} = 0$$

অর্থাৎ, রুদ্ধতাপীয় প্রক্রিয়ায় এনট্রপি পরিবর্তন হয় না।

### Change of Entropy

1) change of entropy in reversible adiabatic process :- (রুদ্ধতাপীয় প্রক্রিয়া)

By definition, the heat absorbed in a  
 reversible adiabatic is zero,

$$dQ_1 = 0$$

Hence for such a process entropy change

is given by,



$$ds = \frac{dQ_r}{T} = 0$$

where,  $S = \text{Constant}$

As the  $S$  is a constant and such process is called isentropic.

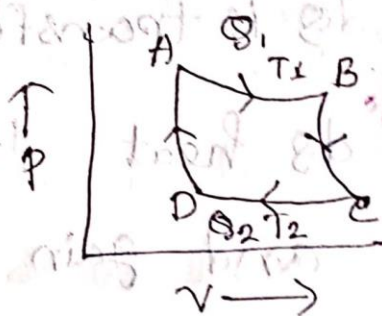
~~Change~~ Change of Entropy in

Reversible thermal process :-

(প্রতিবর্তী প্রক্রিয়ায় এনট্রপির পরিবর্তন)

In reversible isothermal process the temperature remains constant and for such a process

$$S \approx \int \frac{ds}{T}$$



In AB,

The entropy increases =  $\frac{Q_1}{T_1}$

In CD,

$$= \frac{Q_2}{T_2}$$

$$\therefore \text{Entropy changes, } \Delta S = \frac{Q_1}{T_1} - \frac{Q_2}{T_2}$$

But in reversible process,

$$\Delta S = \frac{Q_1}{T_1} - \frac{Q_2}{T_2}$$

But in reversible,  $\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$

$$\therefore \Delta S = \frac{Q_1}{T_1} - \frac{Q_1}{T_1} = 0$$

$\therefore$  Entropy didn't change.

changes of Entropy in irreversible process:

Let ~~to~~ the temperatures of two objects are respectively  $T_1$  and  $T_2$ . If  $T_1 > T_2$  then heat will flow from the warm object to the cold object.

$dQ$  is transferred from warm to cold one.

$dQ$  heat is lost by warm object

and gain by cold object,

Therefore,

$-\frac{dQ}{T_1} =$  Decrease of entropy of warm object

$\frac{dQ}{T_2} =$  Increase of entropy of cold object.



Changes in Entropy,  $\Delta S = \left(-\frac{dq}{T_1}\right) + \left(\frac{dq}{T_2}\right)$

Since,  $T_1 > T_2$

$\Delta S > 0$

So, irreversible entropy is always positive.

Heath Death of the Universe

Everything in nature tries to acquire the state of equilibrium (समत).

As the systems, or when objects stop sharing temperature between them is called as thermal equilibrium.

As the systems proceed towards the equilibrium their entropy also increases. The entropy of a system becomes maximum when we do not get any work from it.

Since everything in nature wants to attain equilibrium so the entropy of nature is

gradually ~~increases~~ increasing  
of the universe  
So, the entropy  $\uparrow$  when it will attain the  
highest point then everything will attain  
the same temperature.

As a result heat energy will not be  
possible to convert into mechanical energy.

It is said to be as the heat death of  
the universe.

### The Third Law of Thermodynamics

"The entropy of a system approaches a  
constant value as its temperature approaches  
absolute zero."

This is because a system at zero temperature  
exists in its ground state, so that  
its entropy is determined only by the  
degeneracy of the ground state.

(अवस्था)  
↓  
(अवस्था)



### Math Solve

① A quantity of air @ at 27 C and atmospheric pressure is suddenly compressed to half its original volume. Find the final pressure and temperature.

=> Here,

Initial pressure,  $P_1 = 1 \text{ atm}$

Initial Temperature,  $T_1 = 27 \text{ C}$

$$= (27 + 273) \text{ K} \\ = 300 \text{ K}$$

$$\gamma = 1.4$$

Let,

Initial volume,  $V_1 = V$

Final volume,  $V_2 = \frac{V}{2}$

Final pressure,  $P_2 = ?$

Final Temperature,  $T_2 = ?$

=> We know that,

During sudden compression the process is adiabatic.

Hence,

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\text{OR, } P_2 = \frac{P_1 V_1^\gamma}{V_2^\gamma}$$

$$\text{OR, } P_2 = \left( \frac{1 \times V}{\frac{V}{2}} \right)^{1.4}$$

$$= (2)^{1.4} = 2.639 \text{ atm}$$

Again,

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\text{OR, } T_2 = T_1 \left( \frac{V_1^{\gamma-1}}{V_2^{\gamma-1}} \right)$$

$$\text{OR, } T_2 = 300 \times \left( \frac{V}{\frac{V}{2}} \right)^{1.4-1}$$

$$\text{OR, } T_2 = 300 \times \left( \frac{V}{\frac{V}{2}} \right)^{0.4}$$

$$\text{OR, } T_2 = 300 \times (2)^{0.4}$$

$$= 395.852 \text{ K} = 122.85^\circ \text{C}$$

(Ans)



Problem-2 :-

Find the efficiency of the Carnot's engine working between the steam point and the ice point solution :-

$\Rightarrow$  Here,

Initial Temperature,  $T_1$  (Steam point) =  $100^\circ\text{C}$

$$= (100 + 273) \text{K} = 373 \text{K}$$

Final Temperature,  $T_2$  (Ice point) =  $0^\circ\text{C} = (0 + 273) \text{K}$

Efficiency,  $\eta = ?$

we know that,

$$\eta = 1 - \left( \frac{T_2}{T_1} \right)$$

$$= 1 - \frac{273}{373}$$

$$= \frac{100}{373}$$

$$\therefore \eta = \frac{100}{373} \times 100\% \approx 26.8\%$$

(Ans)

Problem-03:-

A Carnot's engine whose temperature of the source is 400K takes 200 calories of heat at this temperature and rejects 150 calories of heat to the sink. What is the temperature of sink? Also calculate the efficiency of the engine?

⇒ Here,

$$\text{Initial heat, } Q_1 = 200 \text{ cal}$$

$$\text{Final heat, } Q_2 = 150 \text{ cal}$$

$$\text{Initial Temperature, } T_1 = 400 \text{ K}$$

$$\text{Final Temperature, } T_2 = ?$$

⇒ we know that,

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \quad \text{or, } T_2 = \frac{T_1 \cdot Q_2}{Q_1}$$

$$\text{or, } T_2 = \frac{400 \times 150}{200} = 300 \text{ K}$$



Again,

$$\begin{aligned}\text{Efficiency, } \eta &= 1 - (T_2/T_1) \\ &= 1 - (300\text{K}/400\text{K}) \\ &= \frac{1}{4} = 0.25 \times 100\% \\ &= 25\%\end{aligned}$$

Ans)

(4) If a system absorbs 1100 j heat and work is done 300 j on the system, Find the internal energy of the system.

$\Rightarrow$  Here, Heat,  $Q = 1100\text{ j}$

work done,  $W = 300\text{ j}$

Internal energy,  $U = ?$

We know that,

$$U = Q - W$$

$$= (1100 - 300)\text{ j}$$

$$= 800\text{ j}$$

(Ans)